This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

HDI
HW 1WY9 W

Physical Measurements

T. C. McKay

KE 3046

HARVARD UNIVERSITY

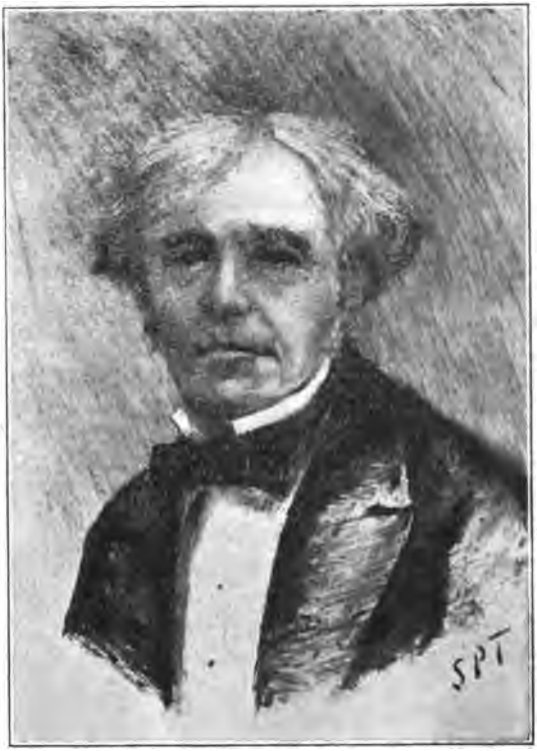
LIBRARY OF THE
JEFFERSON PHYSICAL
LABORATORY

GIFT OF
WALLACE CLEMENT SABINE

FROM THE LIBRARY
OF
PROFESSOR W. C. SABINE

With the compliments of

J. C. McKay



Michael Faraday

PHYSICAL MEASUREMENTS

IN

SOUND, LIGHT, ELECTRICITY AND MAGNETISM

BY

THOMAS C. McKAY, PH. D.
Instructor in Physics in the University of California

FORMERLY
Austin Teaching Fellow at Harvard University

BERKELEY
1908

KE 3046

HARVARD
UNIVERSITY
LIBRARY

4-3x41

Printed by The Stanley-Taylor Company San Francisco

INTRODUCTION

This book is primarily intended as a text-book in physical measurements and a laboratory manual for second year students in physics at the University of California. At this institution there is offered to first year students a laboratory course in mechanics, properties of matter, and heat. The subjects considered in this book are therefore Sound, Light, Electricity, and Magnetism. The experiments here described have been chosen for the most part because they have proved to be valuable exercises in elementary laboratory courses given by instructors in physics at the University of California. A number of the experiments in this book have been described by Professor Drew, now of Leland Stanford University. Professor Hall of the University of California and Professor Minor of the University of Nevada have also contributed greatly to the development of the course. The writer has drawn upon several years experience at the University of California, as well as upon his previous experience at Harvard University, in the endeavor to select the most valuable experiments for an elementary laboratory course in the subjects of which this book treats. As compared with the books issued by the writer's predecessors the sections on Electricity and on Sound have been enlarged. In the treatment of the subject of Electrostatics the writer is indebted to Professor Noack for his careful descriptions in his *"Elementare Messungen aus der Electrostatik."* Considerable use has also been made of Poynting and Thomson's *"Sound."* A description of the principal types of sensitive galvanometers has been given in the section on Electricity, with a short discussion of the conditions of greatest sensitiveness for these instruments.

It is recommended as the result of the experience of various instructors (in the course for which this book was written) that students should perform the experiments individually rather than in groups. If in large sections (and particularly in experiments that require a large number of measurements) it be necessary that some should work in pairs, care should be taken that each student familiarize himself with each kind of measurement involved. Calculations can in general be best performed during the laboratory period. It is easier to make the calculations with the apparatus set up. Frequently too it will be found that one requires some important datum that has not been noted, or that some essential observation is obviously faulty. It is desirable to plot the results of observations wherever possible. For this purpose as well as for the arithmetical calculations notebooks with cross-ruled paper are very convenient.

While nearly all the numbered exercises in the book require the measurement of some physical quantity, there is still opportunity for the student to do some qualitative work. In the exercise on the Wheatstone bridge, for example, there are provided a sensitive galvanometer, a resistance box, a mounted bridge wire, and some specimens whose resistance is to be tested. To see how the result of the measurement may be affected by various conditions one may place the finger on the junction of two dissimilar metals included in the bridge and notice the effect on the galvanometer when the latter is connected in the usual position. He may try likewise the effect of warming a portion of the bridge wire or again the effect of introducing self-induction into one of the arms of the bridge. As the result of such an investigation it is probable that the reasons for the ordinary precautions in the comparison of resistances would become clear. The construction of the galvanometer and of the resistance box are also fair subjects of enquiry if pursued with due regard to the future usefulness of the instrument. The writer has made many suggestions regarding the care of the more valuable pieces of apparatus. It is hoped that the student will read these carefully before working with such apparatus. It is the writer's custom to assign experiments several days in advance so that it is possible for the student to prepare himself by reading in advance.

If the student is to get the best results from a laboratory course it is well to exercise great care in drawing conclusions. The most valuable part of an exercise usually consists in the thorough understanding of the reasons for the various precautions that are necessary for a good measurement. In his text-book on physics for secondary schools Professor Slate has shown that critical studies of great value may be made in very elementary laboratories. Precautions that have been taken in securing any result should be recorded in the notebook. In some cases it may be easy to form an estimate of the possible percentage error in the result of a measurement.

The writer is in the habit of giving an opportunity for special work to students who may become interested in any special problem after they have performed in a satisfactory manner the most fundamental experiments in the course. Such students are furnished with the necessary apparatus so far as the resources of the laboratory permit.

The writer is indebted for suggestions made by Professor E. E. Hall, who kindly looked over the manuscript. Mr. Harold Edwards has suggested improvements in some of the apparatus and has typewritten some of the manuscript. The writer has received much assistance from his wife in the preparation of the drawings from which the cuts have been made.

THOMAS C. MCKAY.

Berkeley, August, 1908.

TABLE OF CONTENTS

PAGE

Introduction iii

SOUND.

I. MEASUREMENT OF VELOCITY OF SOUND, FREQUENCY OF VIBRATIONS, AND ELASTICITIES OF MATERIALS.

The Ear.—Measurement of the Velocity of Sound in Different Media.—Experiment 1: Frequency of the Fundamental Vibration of a Tuning-Fork.—Estimation of Very Short Intervals of Time with the Chronograph.—Experiment 2: Frequency of the Fundamental Note of an Organ Pipe.—Use of the Siren.—Experiment 3: Measurement of Frequency by means of Stationary Waves.—Waves in Solids.—Velocity of Sound-Waves in Rods.—Experiment 4: Determination of Young's Modulus for the Material of a Wire.—Velocity of Sound-Waves in Rods in which the Radial Motion is Not Negligible.—Experiment 5: Velocity of Longitudinal Waves in Rods.—Kundt's Tube.—Adiabatic Elasticity of Gases.—Experiment 6: Velocity of Sound in Various Gases.—Ratio of the Specific Heats.—Relation of the Ratio of the Specific Heats to the Structure of the Molecule.—Transverse Vibrations of Strings.—Experiment 7: Measurement of Tension in a String by Acoustical Methods.—The Sonometer.—Determination of Pitch by Comparison with a Standard Tuning-Fork.—Comparison of Nearly Equal Notes by means of Beats.—Experiment 8: Determination of the Frequency of an Alternating Current with the Aid of the Sonometer.—Stroboscopic Method of Measuring Frequencies.—Illumination of Sound-Waves.—Questions................ 5

PAGE

II. DIFFRACTION AND INTERFERENCE EXPERIMENTS IN SOUND.—ABSORPTION IN ROOMS.

Diffraction.—Interference.—Experiment 9: Quincke's Interference Apparatus.—Koenig's Manometric Flame.—Interference Experiments in Unconfined Air.—Conduction of Sound.—Absorption of Sound in Rooms.—Questions 34

III. COMPOSITION OF VIBRATIONS.

Experiment 10: The Pendulum.—Experiment 11: Combination of Vibrations at Right Angles by means of the Harmonograph.—Experiment 12: Simple Wave Form.—Combination of Waves. 42

LIGHT.

I. DIFFRACTION AND INTERFERENCE OF LIGHT-WAVES.

General Properties of Ether-Waves.—Experiment 13: Diffraction Patterns and Interference Bands.—Experiment 14: Measurement of Wave-Lengths by means of Young's Interference Bands.—Theory of Young's Experiment.—Experiment 15: Measurement of Wave-Length of Sodium Light by means of Newton's Rings.—Questions. 46

II. MEASUREMENT OF REFRACTIVE INDEX.

General Relations.—Experiment 16: Preliminary Measurements of Refractive Index and Critical Angle.—Experiment 17: Determination of Refractive Index by Measurement of Apparent Displacement through a Plane-Parallel Plate.—Refraction through a Thin Lens.—Refraction through a Prism at the Angle of Minimum Deviation.—Experiment 19: Measurement of Critical Angle and Refractive Index with Pulfrich's Refractometer.—Questions. 56

III. STUDY OF CONSTITUENT PARTS OF OPTICAL INSTRUMENTS.

Experiment 20: Measurement of Radii of Spherical

PAGE

Surfaces with the Spherometer, (*a*) The Ring-Spherometer, (*b*) The Peg-Spherometer.—Tests of Plane Surfaces by means of Interference Bands from Monochromatic Light.—Experiment 21: Measurement of Radius of Curvature of Spherical Surfaces by Optical Methods, (*a*) Large Convex Mirror, (*b*) Small Convex Mirror, (*c*) Concave Mirror.—Experiment 22: Measurement of the Focal Length of a Thin Lens by the Method of Conjugate Foci, Spherical Aberration, Chromatic Aberration.—Experiment 23: Measurement of the Focal Length of a Divergent Lens, (*a*) Direct Measurement, (*b*) Close Combination with a Convergent Lens, (*c*) Combination with a Convergent Lens which is at the Distance *d* from the Divergent Lens.—Experiment 24: Study of the Terrestrial Telescope.—Experiment 25: Measurement of the Angles of a Prism by means of a Spectrometer.—Measurement of Refractive Index of Material of Prism for Light of Different Frequencies.—Experiment 26: Calibration of a Spectroscope by means of Arc Spectra.—Experiment 27: Determination of the Wave-Lengths of Fraunhofer Lines by means of the Spectrometer and Grating.—Photometry.—Experiment 28: Measurement of Candle-Power with a Bunsen Photometer.—Experiment 29: Measurement of Absorption by Different Materials for Light of a Particular Wave-Length. — Measurement of Reflecting Power.—Application of Photometry to the Measurement of High Temperatures.—The Photographic Objective.—Achromatic Combinations of Lenses. 68

IV. POLARIZED LIGHT.

Experiment 3: Plane-Polarized Light by Reflection.—Measurement of Angle of Complete Polarization.—Double Refraction in Calcite.—Experiment 31: Study of Circularly Polarized Light and Elliptically Polarized Light.—Experiment 32: Rotation of the Plane of Polarization by means of Quartz Crystals cut Perpendicularly to the Axis.—Saccharimetry.—Questions. 103

PAGE

ELECTRICITY AND MAGNETISM.

I. FUNDAMENTAL EXPERIMENTS IN ELECTROSTATICS.
Quantity, Potential, Capacity.—Experiment 33: Use of Gold-Leaf Electroscope, (*a*) Methods of Charging, (*b*) Causes of Leakage, (*c*) Distribution of Charge on a Conductor, (*d*) Distribution of Potential around a Charged Coiductor.—Experiment 34: Calibration of an Electroscope.—Measurement of Capacity.—Experiment 35: Measurement of Spark Potentials with the Bichat-Blondlot Absolute Electrometer.—Questions. 111

II. MEASUREMENT OF MAGNETIC MOMENT AND OF FIELD STRENGTH.
Experiment 36: Determination of the Ratio of the Magnetic Moment of a Magnet to the Intensity of the Horizontal Component of the Earth's Field.—Experiment 37: Determination of the Product MH.—Bifilar Suspension.—Experiment 38: Comparison of the Values of H in Different Parts of the Laboratory, (*a*) The Variometer, (*b*) The Magnetic Pendulum.—The Tangent Galvanometer.—The Copper Voltameter.—Experiment 39: Determination of H at the center of the Coil of a Tangent Galvanometer. 122

III. MEASUREMENT OF CURRENT AND RESISTANCE.
Experiment 40: Measurement of Quantity of Electricity with the Gas Voltameter.—Experiment 41: Absolute Measurement of Resistance.—Sensitive Galvanometers. 141

IV. COMPARISON OF RESISTANCES.
Care of Resistance Boxes.—Experiment 42: Comparison of Resistances by means of the Wheatstone Bridge.—Specific Resistances.—Experiment 43: Measurement of Temperature Coefficient of Resistance.—Experiment 44: Making of Resistance Coils with a Carey Foster Bridge.—Experiment 45: (*a*) Measurement of Galvanometer Resistance by Thomson's Method, (*b*) Measurement of Battery Resistance by Mance's Method.—The Daniell Cell.—Experiment 46: Measurement of Conductivity

PAGE

of Electrolytes with Alternating Current and Telephone.—Degree of Dissociation.—Experiment 47: Tests on Water by means of Conductivity Measurements.—Experiment 48: Measurement of Very Low Resistances.—Experiment 49: Measurement of Very High Resistances.—Calibration of a Sensitive Galvanometer.—Questions. 153

V. MEASUREMENT OF ELECTROMOTIVE FORCE.
Standard Cadmium Cell.—Experiment 50: Comparison of Electromotive Forces by means of the Potentiometer.—Internal Resistance of a Daniell Cell.—Experiment 51: Measurements of Thermal Electromotive Forces.—Thermoelectric Diagram, Neutral Point.—Causes of Inconstancy of Electromotive Force in Galvanic Cells.—Experiment 52: Measurement of Electromotive Force of Polarization.—Commercial Storage Cell.—Experiment 53: Electromotive Force in a Conductor Revolving in a Uniform Field.—Angle of Dip. 175

VI. COMPARISON OF MAGNETIC FIELDS. COMPARISON OF SPECIFIC INDUCTIVE CAPACITIES.
Ballistic Galvanometer.—Experiment 54: Measurement of the Field in the Air Gap of a Horseshoe Magnet.—Intensity of Magnetization in a Bar Magnet.—Experiment 55: Measurement of Specific Inductive Capacity.. 187

SOUND

I. AUDITORY APPARATUS OF THE EAR — MEASUREMENT OF VELOCITY OF SOUND, FREQUENCY, WAVE-LENGTH, YOUNG'S MODULUS, ADIABATIC ELASTICITY OF GASES, RATIO OF THE SPECIFIC HEATS, ETC.

Measurements in sound deal principally with the propagation of the disturbances that cause the corresponding sensation. The measurement of the velocity with which these disturbances pass through materials, affords a convenient means of determining Young's modulus and the ratio of the specific heat at constant pressure to the specific heat at constant volume. These are important constants in practical engineering. We shall first give a brief description of the auditory apparatus of the ear.

The Ear. Sound-waves from the air are collected by the outer ear or auricle and pass into the auditory canal which terminates in the drum or *membrana tympani.* The drum is, when stretched, in the form of a conical hood, the base adhering to a ring of bone while the apex is attached to a bone called the hammer. Two other bones called the anvil and stirrup form with the hammer and drum a system of levers by means of which the vibrations of the drum are transmitted to the end of the stirrup with less amplitude but with greater force. The stirrup presses on the *fenestra ovalis* through which the movements of the drum are transmitted to the fluids in the labyrinth of the inner ear. The terminal apparatus of the auditory nerve is immersed in a fluid called the endolymph. One division of the auditory nerve ends at the organs of Corti, which are situated on the basilar membrane. Helmholtz has suggested that the segments of the basilar membrane may be stretched radially with various tensions so that these segments correspond to violin strings of different lengths and tensions. These are possibly capable of sympathetic vibrations. Hence may arise the power which some musicians possess of analyzing complex notes into their primary components. The ear is unsuitable for detecting sound-waves of greater frequency than about 30,000 complete vibrations a second. A suspended mica vane may be used to detect sound of greater frequency than can be detected by the human ear. The latter is also unable to detect the note given by a tuning-fork whose frequency of vibration is much less than about thirty a second. This is the case even when the amplitude of vibration of the prongs of the fork is very great. The ear is very sensitive to sound-waves of frequencies between 1000 vibrations a second and 3000 vibrations a second. An account of an attempt by Lord Rayleigh to find the smallest intensity that could be detected for sound-waves of frequency 2730, will be found in Poynting and Thomson's *Sound*.[1] Students interested in the auditory mechanism of the ear will find it discussed at length in Helmholtz's *Sensations of Tone*.[2]

Measurement of the Velocity of Sound in Different Media. The velocity of sound-waves depends for a particular fluid material chiefly on the bulk elasticity and the temperature, but also to a slight extent on the intensity of the sound. The velocity of sound can easily be measured directly in the case of air and water by noting the time required for the disturbance to pass over a certain distance. In the case of air the velocity has been frequently measured by noting the interval between the flash of a distant cannon (as seen by the observer) and the arrival of the report. The velocity of light is so great in comparison with that of sound that no correction is necessary for the time required for the light to pass from the cannon to the observer. It is necessary however to note the speed and direction of the wind and also to form an estimate of the mean temperature of the air through which the sound is carried. Comparison of results, obtained between stations situated on mountain tops and again between stations at lower levels, shows that the velocity of sound in air is independent of the pressure between wide limits of pressure. Observations made by Greely in the polar regions gave for the velocity of sound in meters per second at $t°$ C.

---

[1] Edition of 1900, p. 118.

[2] Translation of Third German Edition by Alexander Ellis, 1875, pp. 189-226.

$$v = 333 + 0.6\,t.$$

This gives $v = 333\left(1 + \frac{0.6}{333}t\right) = 333\left(1 + \frac{1}{555}t\right)$. The temperature coefficient of the velocity of sound at 0° C. is given by Greely's observations as about one half the corresponding coefficient of increase of pressure in the constant-volume air thermometer, where the pressure is of course proportional to the absolute temperature. Greely's observations therefore suggest that the velocity of sound in air is proportional to the square root of the absolute temperature. This is fully confirmed by experiment.

The errors due to the wind may be nearly eliminated by firing cannon (alternately from the two stations) a large number of times. The mean of the two periods observed for the passage of the sound in the two opposite directions is then taken as the period during which the sound would travel between the two stations in the undisturbed air. A correction has also to be made for the humidity. This increases the velocity of the sound-waves by lessening the density of the air. If v' and ρ' be the respective values of the velocity of sound and the density of the air under the conditions of the experiment and v and ρ the corresponding values for dry air at the same temperature, then

$$\frac{v}{v'} = \sqrt{\frac{\rho'}{\rho}}$$

The law which connects all the above-mentioned relations may be put in the form

$$v = \sqrt{\frac{\mathrm{P}k}{\rho}},$$

where P is the pressure, k the ratio of the specific heat of the air at constant pressure to its specific heat at constant volume, and ρ the density.

The velocity of sound in air increases slightly with the intensity, but approaches a limiting value as the intensity decreases. Regnault found that over a distance of 1280 meters from a certain source of sound the mean velocity reduced to 0° C. was 331.4 meters per second, while over a distance of 2445 meters the velocity was 330.7 meters per second. This experiment indicated therefore a decrease of velocity as the intensity decreased. We may take as an approximate value for the velocity of sound at 0° C. in dry air $v = 332$ meters per second. This value may be used in determining the velocity of sound in other materials by comparison with the velocity of sound in air. In the ordinary moist air of the laboratory the velocity is approximately $v = 332\sqrt{1 + 0.004\,t}$, where t is the temperature in ordinary centigrade degrees.

Similar experiments have been made to determine the velocity of sound in water. Its value seems to depend much more on the intensity of the sound than in the case of air. Threlfall and Adair obtained for sea-water the value 1730 meters per second when 9 oz. of gun-cotton were exploded about 160 meters from the receiver. A velocity of 2010 meters per second was obtained when 64 oz. of gun-cotton were exploded at a similar distance from the receiver.

The velocity of sound in substances which are not at hand in large quantities in a homogeneous state may be determined most conveniently by Kundt's method, which will be described in the study of longitudinal vibrations in solids. We shall first consider methods of measuring frequency and wave-length.

Experiment 1. Measurement of the Frequency of the Fundamental Vibration of a Tuning-Fork. A slight style of bamboo (or of stiff paper) is attached to the end of a prong of the fork with a small piece of soft wax. It is desirable to use a rather heavy fork so that the addition of the style will not change the period much. After the base of the fork has been clamped in the desired position, a piece of smoked glass or of smoked paper suitably mounted is used to obtain a record of the vibrations of the fork. To smoke the glass or paper a Bunsen burner with fish-tail opening is used, the smoky flame being applied long enough to obtain a light coating. If the smoked glass is used, it is drawn along under the vibrating fork, while the time is recorded by means of a style fastened to the bob of a pendulum that swings back and forth near the vibrating fork. The pendulum style should barely touch the smoked glass at the middle point of the pendulum swing.

A more satisfactory record may be obtained by mounting smooth paper on the cylindrical surface of a revolving drum. The paper is smoked as previously described. To mark off the time intervals on the record a style may be attached to the armature of a Morse sounder for example, the circuit through the sounder being completed at regular intervals by means of a pendulum bob that makes contact with a drop of mercury at the middle point of its swing. The record given by the fork and the sounder will then appear somewhat like that in figure 1. The record given by the fork is a typical wave form. The curve is a sine curve such as may be obtained by plotting the sines of angles as ordinates, while the angles themselves are plotted as abscissæ. E_1, E_2, E_3, etc., indicate the regular intervals of the passage of the pendulum through the middle point of its swing.

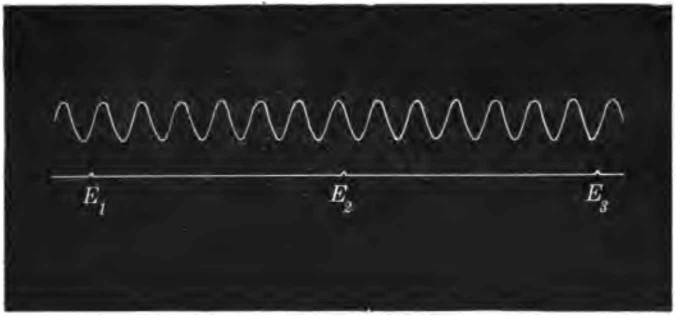


Fig. 1. Record of Tuning-Fork on Sheet Mounted on Revolving Drum with Time Record Given by Pendulum.

If the period of the pendulum be determined by comparison with a chronometer it is easy to calculate the frequency of the fork at the temperature of the room with the given style. Two observers may be required in making this comparison, if the click of the sounder is not sufficiently sharp. Times are noted as the pendulum bob makes contact with the drop of mercury. The record of the comparison may conveniently be made as indicated below.

Time of Middle Point of Swing			**Mean Epoch**			**Time of Middle Point of Swing**			**Mean Epoch**		
H.	**M.**	**S.**	**H.**	**M.**	**S.**	**H.**	**M.**	**S.**	**H.**	**M.**	**S.**
						Interval (2) 50 Swings					
3	23	26.2				3	24	29.3			
		27.1	3	23	27.17			30.5	3	24	30.47
		28.2						31.6			
Interval (1) 10 Swings											
3	23	38.1									
		38.9	3	23	39.03						
		40.1									

For the twelve swings, between the first two mean epochs recorded, the period obtained is 0.9883 second; for 52 swings between the last two mean epochs, 0.9896 second. This serves as a check on the counting of the 50 swings, for if one had been omitted in the counting it would have made a difference of 2 percent in the calculation of the period. The best result is obtained by subtracting the first mean epoch from the last and dividing by the number of intervals, that is by 64. This gives for the period of a swing 0.9891 second. The period of a complete vibration of the pendulum is twice as great.

The temperature of the room should be noted. McLeod and Clarke found for the temperature coefficient of a steel fork the equation

$$n_t = n_o \ (1 - 0.00011 \ t),$$

where n_t is the frequency at $t°$ C. and n_o the frequency at 0° C.

The record of the fork and of the pendulum should be preserved in the note-book. If it has been taken on smoked glass, it may be printed on blueprint paper.

Determine the frequency of one of the steel forks in the laboratory (fundamental note) and calculate the frequency that it would have at 0° C.

Estimation of Very Short Intervals of Time with the Chronograph. An arrangement similar to that above may be used as a chronograph where the frequency of the fork is known, and very short periods of time may be measured in terms of the vibration period of the fork. The style of the tuning-fork is sometimes made of a fine metal wire and an electric spark passed through the paper to the drum at the beginning and end of the interval that it is desired to measure. A pendulum with electrical connections makes a parallel time record close to that of the fork. Such a method of measuring short intervals of time is frequently used in the psychological laboratory in measuring for example the interval between the exciting cause and the reflex action of the subject. If the frequency of the fork be 100 vibrations a second and if it be possible to read the record to one fourth of a vibration period, the time interval may be estimated to 0.005 second.

Experiment 2. Measurement of the Frequency of the Fundamental Note of an Organ Pipe. Use of the Siren. An account of the siren of Cagniard de la Tour is given in Watson's *Physics* (p. 381, 1903 edition). This form of siren has a wind chest covered with a stationary disk that has been pierced regularly (on the circumference of a circle whose center lies in the axis of the disk). In contact with this is another disk movable around the axis of the siren. It is pierced in the same way as the stationary disk except that its openings make an angle with those of the stationary disk. When air is pumped into the wind chest it will pass out whenever the openings of the movable disk coincide with those of the stationary disk. The change of direction of the air in passing from one disk to the other means a change of momentum of the issuing air that carries the movable disk around in a definite direction of rotation with respect to the other disk. If there are p openings in each of the disks and m revolutions a second of the revolving disk then the frequency of the note given by the siren is mp.

The siren as used by Helmholtz had a *number of rows* of openings at different distances from the axis of the movable disk. The latter was turned by means of an electric motor. The air stream was obtained by means of a stiff paper turbine attached to the disk. By means of a number of stops it was possible to use any desired one of the rows of openings. The speed of the motor could be easily regulated by means of a resistance in series with it. In this way a much more uniform note can be obtained than when the bellows are used as the source of power. In the latter case it is necessary to control the speed of revolution of the siren by placing the finger lightly against the axle of the movable disk, in tuning the siren to the organ pipe. Another observer obtains the number of revolutions of the disk (with the aid of the counter attached to the axle) during a convenient period. The accuracy of the tuning of the two sources may be tested by counting the beats between the two notes. When the interval between the successive beats lengthens out until the beats can no longer be distinguished, the speed counter is read. Some difficulty in tuning is sometimes experienced from the fact that the note of the organ pipe is usually a combination of the fundamental and some of its harmonics, while the siren gives a pure note. Close attention to the fundamental note of the organ pipe is therefore required.

The temperature and humidity of the air should be noted. If ρ be the density of the air under the conditions of the experiment and ρ_0 the density of dry air at 0° C., then the corresponding frequencies n and n_0 will be related by the equations

$$\frac{v}{v_0} = \frac{\lambda . n}{\lambda . n_0} = \sqrt{\frac{\rho_0}{\rho}},$$

where v represents velocity of the sound in air and λ its wave-length in air. The latter bears a fixed relation to the length of the pipe if we use only the fundamental note. Therefore

$$\frac{n_0}{n} = \sqrt{\frac{\rho}{\rho_0}}$$

Determine the frequency of one of the organ pipes in the laboratory, reducing your results to the conditions of dry air whose temperature is 0° C.

Experiment 3. Measurement of Frequency by means of Stationary Waves. A note is sounded in front of a tube having a movable piston. In this case periodic disturbances are set up at the mouth of the tube by the vibrating source. If the length of the tube be such that an inward movement of the air is reflected back from the surface of the piston in time to help a succeeding outward movement (due to a motion of the vibrating source away from the tube), then the tube will sound a note in resonance with the vibrating source. This will evidently be the case for example when the distance from the mouth of the tube to the piston and back is travelled in one half a vibration period of the fork. At the beginning of this interval the air at the mouth of the tube is forced inwards, at the end of the interval it is forced outwards. The forced vibration then agrees with the natural vibration of the air in the tube of given length. When the distance to the piston and back then is one half a wave-length, the period of free vibration of the tube coincides with the forced vibration at the mouth, and the latter is reinforced. In this case the distance from the mouth of the tube to the piston is one fourth of a wave-length. We should obtain similar results if this distance were three fourths of a wave-length or any odd integral multiple of a fourth of a wave-length.

While the above statement is approximately true a correction has to be made for narrow tubes in reducing the wave-length to what it is in free air. This is because the velocity of the wave in the narrow tube is changed by friction and heating. The distance from the mouth of the tube to the first position of the piston for resonance is not exactly one third of the distance from the mouth to the second position for resonance owing to disturbances at the mouth of the tube. In more exact measurements therefore the wave-length is measured in the tube by measuring the distances between successive positions of the piston that give resonance. The distance between two such consecutive positions is one half of a wave-length in the tube. According to Kundt a half wave-length l mm. long, observed in a glass tube d mm. in diameter, should be corrected to reduce to the half wave-length in open air by the addition $0.02 \frac{l^{\frac{3}{2}}}{d}$ mm. The data and calculations may be conveniently recorded as follows:

Observed positions of piston for resonance...........
Mean half wave-length l as observed in the tube in mm. ....
Corrected value $l' = l \left(1 + 0.02 \sqrt{\frac{l}{d^2}}\right)$ mm. $=$ .....
Wave-length in free air $\lambda = 2\,l' =$ .... mm., reduced to meters $=$ ....
Temperature of air in tube, t (in centigrade degrees) $=$ ....
Velocity of sound in free air $= 332\sqrt{1 + 0.004\,t} =$ ...
Frequency of note $= \frac{\text{Velocity}}{\text{Wave-Length}} =$............

Waves in Solids. In gases the resistance due to the displacements of particles takes the form of increase of pressure. The sound waves are longitudinal, that is the displacements of the air particles are parallel to the direction of propagation of the wave. The velocity is determined by the bulk elasticity (the value of this for sudden changes of pressure) and the density. In the case of solids on the other hand there is resistance not only to expansion but also to changes of form. Hence in homogeneous continuous solids disturbances may be propagated as longitudinal waves and as transverse waves. The velocity of longitudinal waves in such a solid depends on the bulk elasticity, the rigidity, and the density. The velocity of the transverse waves depends on the rigidity and the density. The displacement is here perpendicular to the direction of the propagation of the disturbance. Light waves are transverse waves in the ether.

In the case of thin rods and wires longitudinal waves may be obtained by rubbing lengthwise with a wet cloth. The study of these longitudinal waves is of great importance because their velocity in a thin rod of a given material depends on the value of Young's modulus for that material.

Velocity of Longitudinal Waves in Rods. Suppose the rod indicated in figure 2 to be of uniform cross-section of area one square centimeter, but very long as compared with its diameter. Let A and B be two points on the rod very near together. Ordinates are erected perpendicular to the rod to represent the displacement (caused by a travelling impulse or wave) at a given instant at the section from which the ordinate is drawn. The curve PQ is drawn through the extremities of the ordinates. Tangents to the curve at P and Q intersect in S. QV and ST are drawn parallel to the axis of the ordinates through P and S, cutting ST and QV in T and V respectively at right angles.

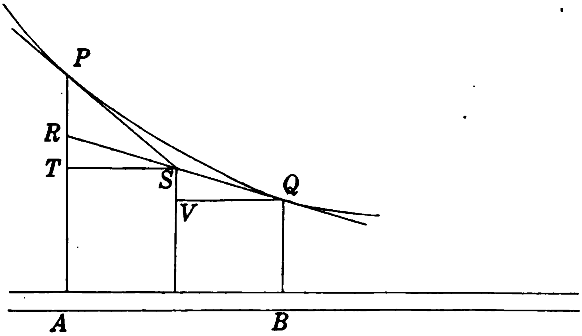


Fig. 2. Displacements in Thin Rod.

Since A and B are very near together on the rod we may take TS = VQ = ½AB. As PS is tangent to the displacement curve at P the ratio $\frac{PT}{TS}$ represents the elongation per centimeter that the rod is experiencing at the section S. This elongation is resisted by a force that is equal to the elongation per centimeter multiplied by the Young's modulus for the material. This force per square centimeter is therefore $\frac{PT}{TS} \times E$. Similarly the elongation per centimeter experienced by the rod at the section B is $\frac{SV}{VQ}$, or producing QS to meet PT in R, is equal to $\frac{RT}{TS}$. The force per square centimeter resisting this elongation at Q is therefore $\frac{RT}{TS} \times E$. These unequal forces at the ends of the element AB leave a resultant force $\left(\frac{PT - RT}{TS}\right) \times E$ acting on the element AB. This force must be equal to the mass of the element $\rho(AB)$ multiplied by the mean acceleration of the element. To find the latter we consider the interval *t* that is required for the impulse or wave to travel from A to B. Let the velocity of the wave be v.

Then the wave travels from A to B in the time $t = \frac{(AB)}{v}$. At the beginning of this interval B is being moved at such a rate that it would go the distance measured by SV in half the time that the wave takes to go from A to B, that is $\frac{t}{2}$. Its initial velocity of displacement is therefore $\frac{SV}{\frac{1}{2}t}$. At the end of the interval t, however, the displacement curve above B is the same as that above A at the beginning of the interval. Therefore the velocity of B's displacement at the end of the interval is $\frac{PT}{\frac{1}{2}t}$. The mean acceleration during the time t is equal to the gain of velocity divided by the time t, that is $\left(\frac{PT - RT}{\frac{1}{2}t}\right) \div t = \frac{PR}{\frac{1}{2}t^2}$. Since A and B are very near together this may be taken as the acceleration of the element AB. Putting then the resultant force on the element equal to the mass multiplied by the acceleration we have

$$\frac{(PT - RT)}{TS} \times E = \rho\ (AB) \times \frac{(PT - RT)}{\frac{1}{2}t^2}, \text{ and}$$

$$E = \frac{\rho \times (TS) \times (AB)}{\frac{1}{2}t^2} = \frac{\rho \times \frac{1}{2}(AB)^2}{\frac{1}{2}t^2} = \rho\left(\frac{AB}{t}\right)^2.$$

But $\frac{AB}{t}$ is the velocity of the wave which we have called v.

$$\text{Therefore} \qquad \left(\frac{AB}{t}\right)^2 = v^2 = \frac{E}{\rho}$$

Experiment 4. Determination of Young's Modulus for the Material of a Wire. A steel wire about ten meters long and one or two millimeters in diameter is clamped at each end by means of two vises. The latter are fastened to a long board. The vises are moved along the board until the wire is under a slight tension. Longitudinal vibrations are set up in it by rubbing the wire with the fingers, which have been touched with powdered resin. At the same time a tuning-fork of frequency $n = 256$ is sounded and the length of the wire adjusted until it gives the same note. The points at which the wire meets the vises are nodes, and the distance between two consecutive nodes is one half of a wave-length. Therefore the velocity $v = 2ln = \sqrt{\frac{E}{\rho}}$, from which Young's modulus may be found if the density ρ is known. The latter may be determined by weighing a specimen of the wire whose length and diameter have been carefully determined or by weighing a piece of the wire in air and again in water.

The harmonics of the fundamental note are usually present in the experiment just described and are often very intense. The student may isolate these to some extent (by for example holding the wire with the fingers at distances from the end such that the whole length is an integral multiple of its shorter part), while another student excites the shorter length by rubbing it lengthwise at its middle point.

The formula given above states that the velocity of a longitudinal vibration is independent of the tension of the wire. If this is the case the note should not be altered when the tension of the wire is varied within wide limits (while the length is kept constant). The student should try this experiment, care being taken not to stretch the wire beyond the elastic limit of its material.

Velocity of Sound-Waves in Rods in which Radial Motion Is Not Negligible. When a rod is stretched there is also a side contraction. The amount of this side contraction bears to the longitudinal extension a ratio called Poisson's ratio that depends on the material. If for example the original normal length of the stretched rod be L and the corresponding radius R, and in the stretched condition the corresponding quantities be $L + l$ and $R - r$, then Poisson's ratio for the material is $\left(\frac{r}{R}\right) \div \left(\frac{l}{L}\right)$. In the case of cork, where extension diminishes the radius only slightly, Poisson's ratio is only a small fraction not much above zero. In the case of jellies and india-rubber where the volume remains nearly constant for small stretching forces Poisson's ratio is nearly ½. For most substances it is from ⅓ to ¼. The velocity of the longitudinal disturbance in a rod of uniform diameter d is diminished by $\frac{p^2 \pi^2 d^2}{4 \lambda^2}$ of the full value as a result of side contractions and expansions of the rod. Here p represents Poisson's ratio and λ represents the wave-length. Hence a wave corresponding to a given note will travel more rapidly than the waves corresponding to its harmonics.

Experiment 5. Comparison of the Velocity of Longitudinal Waves in Rods with the Velocity of Waves of the Same Frequency in Air. Kundt's Tube. A rod (provided with a light piston of cardboard or cork on one end) is fixed firmly at the middle in a clamp (held by a vise). Longitudinal vibrations may be set up in it by stroking either free end with a cloth that has been soaked with alcohol. If a tube whose diameter is larger than the piston be slipped over the latter, while the opposite end of the tube is stopped up, the air in the tube will be set in vibration. The energy of vibration of the stationary waves in the tube depends on the position of the piston. The latter (when the vibrations are strongest) is not situated at a loop as would be expected but between a loop and a node. The positions of the loops and nodes are made evident by scattering inside the tube fine filings of cork, or fine infusorial earth, or lycopodium powder. The infusorial earth is particularly good where the velocity of sound in gases is studied at high temperatures. Students frequently have difficulty in getting the dust figures in a Kundt's tube. If the rod is not firmly clamped the vibrations are communicated to the loose support so that insufficient energy is given to the air. If the piston is loose on the end of the rod a similar cause prevents the dust figures from appearing. The tube should be very dry. It may be dried quickly by drawing through it a cloth that has been warmed with a Bunsen flame. It is easier to obtain the figures in narrow tubes because the extent of dust surface is greater in comparison with the vibrating volume. In this case however the velocity of the wave is lessened by friction and a correction such as Kundt derived must be applied in order to use the value for the velocity of sound in free air.

We have

$$\frac{\text{Velocity of Longitudinal Wave in Rod}}{\text{Velocity of Wave of Same Frequency in Air in Tube}} = \frac{l_r}{l_a},$$

where l_r is the sum of the vibrating lengths (supposed equal) of the rod, and l_a is the mean distance between two consecutive nodes (or loops) in the tube. The velocity of sound in the tube at temperature t is

$$332\ (\sqrt{1 + 0.004\ t}) \div (1 + 0.02 \sqrt{\frac{l}{d^2}})$$

meters per second, where however the l and d in Kundt's correction are expressed in millimeters.

The velocity of sound in the actual rod is reduced to what it would be in a thin rod of the same material by multiplying by the factor $1 + \frac{p^2\, \pi^2\, d^2}{16\, (l_r)^2}$ where p is Poisson's ratio, d is the diameter of the rod and l_r is the length of the vibrating parts of the rod. This correction may be so small that it is negligible in comparison with the errors due to observation of the distances between the nodes in the tube.

Young's modulus for the material is equal to the square of the velocity of longitudinal waves in a thin rod of the material multiplied by the density. It is conveniently expressed in dynes per square centimeter, which it will be if the velocity is expressed in centimeters per second and the density in grams per cubic centimeter.

Determine Young's modulus for some of the following materials: iron, glass, copper, brass, aluminum.

Adiabatic Elasticity of Gases. The adiabatic elasticity of a gas is the value that the bulk elasticity assumes when the compression or expansion is performed in a non-conducting vessel so that no heat can leave or enter during the process. The isothermal elasticity is the elasticity shown by the gas when the operations are carried on in a perfectly conducting vessel with which all portions of the gas are sufficiently in contact so that the heat of compression for example flows out as soon as it appears. If a compression (or expansion) is performed very suddenly however, the gas shows its adiabatic elasticity whether it be in a non-conducting vessel or not, for there is not time for flow of heat between the gas and its surroundings. It is clearly the adiabatic elasticity that we are concerned with in the case of sound-waves, because of the extreme rapidity with which the compressions and expansions of a given element of the material follow each other. In the case of solids and liquids the adiabatic elasticity only slightly exceeds the isothermal elasticity. In the case of gases the ratio of the two elasticities varies from 1 to 1⅔. We shall calculate the value of the adiabatic elasticity of a gram of gas at 0° C. The same form of proof will of course do equally well for any other mass of a given material and for any temperature that is sufficiently far above the critical temperature of the gas.

Let the pressure of the one gram of gas be P dynes per square centimeter and its volume V cubic centimeters at 0° C. Then $PV = 273\,R$, where R is a constant for the given mass of this particular gas. If it now be heated in a vessel with rigid boundaries until its temperature is 1° C., no external work will be done and the amount of heat put into it will be C_v, that is the specific heat at constant volume. If we start again at 0° C., with a portion of the envelope movable so that the pressure remains equal to the outside pressure P (heating the gas again to 1° C.), the amount of heat required will be $C_v + \frac{PV}{273}$, because in addition to having its temperature raised 1 degree as before the gas expands to the amount of $\frac{1}{273}$ of its original volume against the external pressure P. Therefore

$$C_p = C_v + P\frac{V}{273} = C_v + R. \quad (1)$$

Starting again from 0° C., suppose the gas to be compressed in a non-conducting vessel or very suddenly until its pressure rises to $P + p$. The volume will decrease by some amount v (cubic centimeters) and the temperature will rise to some value t° C. The adiabatic elasticity is then the limit approached by the increment of pressure divided by the ratio of the decrease of volume to the original volume (when the compression is made very small), that is the limit approached by $\frac{p}{v/V} = \frac{pV}{v}$. The work of compression may be taken as $\left(P + \frac{p}{2}\right) v$, that is the average pressure multiplied by the change of volume. Since all of this work is used up in heating the gas from 0° C. to t° C., $\left(P + \frac{p}{2}\right) v = (C_v)\,t$ (2). But by the gas law the pressure multiplied by the volume divided by the absolute temperature is the same through all the changes of the gas within wide limits, therefore $\frac{(P+p)(V-v)}{273+t} = \frac{PV}{273}$. (3)

Substituting the value of t from (2) and neglecting the product of the two very small quantities p and v, and subtracting 273 PV from each side of the equation,

$$273\,(pV - Pv) = \left(P + \frac{p}{2}\right) \cdot \frac{v}{C_v} \cdot PV.$$

Therefore the adiabatic elasticity is

$$E = \frac{pV}{v} = P + \frac{\left(P + \frac{p}{2}\right)}{C_v} \frac{PV}{273} = P + \left(P + \frac{p}{2}\right) \frac{R}{C_v}, \text{ from}$$

equation (1).

In the limit $\frac{p}{2}$ becomes negligible in comparison with P. Therefore

$$E = P\left(1 + \frac{R}{C_v}\right) = P\left(1 + \frac{C_p - C_v}{C_v}\right) = \left(\frac{C_p}{C_v}\right) P.$$

By the same arguments that were used in the case of thin rods it may be shown that the velocity of sound in a gas is given by the equation,

$$v^2 = \frac{E}{\rho},$$

where E however is the adiabatic elasticity of volume, whereas in the case of thin rods we had to deal with the adiabatic elasticity of length. Putting for E the value derived above we get

$$v^2 = \frac{P}{\rho} \frac{C_p}{C_v}.$$

It will be a useful exercise for the student to calculate the ratio of the specific heat at constant pressure to the specific heat at constant volume in the case of air, taking v as 332 meters per second at 0° C.

Experiment 6. Comparison of the Velocity of Sound in Other Gases with the Velocity of Sound in Air. Calculation of the Ratio of the Specific Heats. A Kundt's tube somewhat similar to that described for experiment 5 is used here. Precautions have to be taken however to preserve the purity of the gas in the Kundt's tube. At one end of the latter is a movable piston (used in adjusting to resonance). When another gas than air is being used it is desirable to back the piston with cotton wool as a precaution against the entrance of air. At the other end a glass rod or tube (held at its middle by a cork which closes the Kundt's tube) may be used to obtain a high note. Clean glass is very easily excited to longitudinal vibration when rubbed with a cloth that has been soaked in alcohol. Two side tubes have been previously sealed into the Kundt's tube to facilitate the washing out of the air with the gas which it is desired to test. The student will find it convenient to prepare carbon dioxide from marble and hydrochloric acid in a Kipp generator. The carbon dioxide is led from the generator through a wash bottle containing water and then through drying-tubes containing calcium chloride. As carbon dioxide is heavier than air the former is conducted to the Kundt's tube by a side tube which allows the entrance of the carbon dioxide at or near the lowest point of the apparatus while the air is allowed to escape from a tube at the highest part. The change of the gases may be much facilitated by the use of an air pump, in which case care must be taken to shut off connection from the Kipp generator during the exhaustion of the Kundt's tube.

If l_a be the mean distance between consecutive nodes when the tube is filled with air, and if l_x be the corresponding distance when some other gas is used, then $\frac{v_x}{v_a} = \frac{l_x}{l_a}$.

Kundt's correction for the *l*'s when narrow tubes are used may be applied if the accuracy of the work seems to warrant it. If the gas is a simple one, that is not a complex mixture of several gases, the density may be found from the tables. Otherwise the density may be determined by weighing a liter bottle, for example, when full of the gas at a known temperature and pressure, and weighing again when the bottle has been exhausted by a good air pump, and lastly weighing when the bottle is full of water.

The ratio of the specific heats is given as we have seen by the equation

$$\frac{C_p}{C_v} = v^2 \frac{\rho}{P},$$

where v represents the velocity of sound and ρ the density and P the pressure.

Find the ratio of the specific heats for some gas other than air.

Relation of the Ratio of the Specific Heats to the Structure of the Molecule. The method of determining the ratio of the specific heats that has just been described is of importance in some cases to the chemist in determining the number of atoms to a molecule for a given gas. The kinetic theory of gases shows that in the case of gases that have only one atom to the molecule the ratio of the specific heats should be 1⅔. Kundt and Warburg obtained for mercury vapor, which is monatomic, the value 1.66. Boltzmann has given reasons to indicate that gases having two atoms to the molecule should have the specific heat at constant pressure equal to 1⅖ times the specific heat at constant volume. The value found for hydrogen, nitrogen and some other diatomic gases which have been tested varies from 1.40 to 1.41. As the structure of the molecule becomes more complicated we should expect that a greater and greater proportion of the amount of heat required to raise a gram of the gas one degree would be absorbed by the movements of the atoms in the molecule, and that a less proportion would be devoted to increasing the to-and-fro motion of the molecule as a whole. As the molecule becomes more complicated in its structure therefore, we might expect the specific heat at constant volume to become larger with respect to the specific heat at constant pressure. The following table will serve to illustrate these views.

Gas or Vapor	Formula	Number of Atoms in Molecule	$\frac{C_p}{C_v}$
Mercury	Hg	1	1.667
Helium	He	1	1.652
Hydrogen	H_2	2	1.410
Carbon Dioxide	CO_2	3	1.264
Ethylene	C_2H_4	6	1.187
Ether	$C_4H_{10}O$	15	1.079

In the case of helium the determination of the ratio of the specific heats was necessary to the determination of its atomic weight, for it was not known to form any compounds.

Transverse Vibrations of Strings. In Watson's *Physics*[1] the student will find the derivation of an expression for the velocity of a transverse disturbance in a string $v = \sqrt{\frac{T}{m}}$ where T is the force with which the string is stretched and *m* is the mass per unit length of the string (or the linear density of the string). In order to get an expression which shall involve the mass per unit volume we may write the above equation as follows:

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{\frac{T}{A}}{\frac{m}{A}}} = \sqrt{\frac{\frac{T}{A}}{\rho}}$$

---

[1] Page 364, 1903 edition.

Here $\frac{T}{A}$ is the stretching force per unit area of cross-section and ρ is the density of the material, that is the mass per unit volume. The formula now corresponds to that for longitudinal vibrations in a string, the applied stretching force per square centimeter taking the place of Young's modulus, which also represents a force per unit area. In all other respects the formulæ for the velocities of the two kinds of waves are the same.

Experiment 7. Measurement of Tension in a String by Acoustical Methods. The Sonometer. The sonometer used in class is like that shown in Watson (p. 398, 1903 edition). A spring balance is however inserted between the pulley and the nearest wedge so that the pull of the spring is nearly in line with that of the vibrating string. The fundamental note given by the latter is compared with a tuning-fork of rather low pitch (192 to 256 complete vibrations a second). By varying the distance between the wedges when the string is under a suitable tension the student will soon be able to detect beats between the note from the fork and that from the string. When the interval between consecutive beats has lengthened out until they can no longer be detected, the length of the vibrating string is measured with a meter rod and the reading of the spring balance is taken. The latter should be taken several times, the balance being disturbed frequently with the hand to avoid the errors due to friction. Steel pianoforte wires may be conveniently used, one about two millimeters in diameter and one about 0.5 millimeter in diameter. If the densities of the materials composing these wires is the same the data from both may be combined as follows:

Frequency of tuning-fork in complete vibrations per second..........................

Density of material of string in grams per cubic centimeter..........................

Length of Vibrating String	**Readings of Spring Balance (Corrected)**	**Mean Area of Cross-Section of String**	**Stretching Force per Unit Area of Cross-Section**	
			Observed	**Calculated**

The second last column refers to the stretching force per unit area as obtained from the observations with the spring balance and the micrometer calipers (used in measuring the diameter). The stretching force per unit area is *calculated* from the equations $\frac{T}{A} = v^2 \rho$ and $v = 2\, l\, n$, where l is the length of the vibrating string and n the frequency of the fork.

In the calculations it is convenient to use the centimeter-gram-second system throughout, the velocities being expressed in centimeters per second and the results of the last two columns above in dynes per square centimeter. To exhibit graphically the relation between the length of the vibrating wire and the stretching force per unit area of cross-section the corresponding numbers under the first and fourth columns above should be plotted on coordinate paper. In drawing a smooth curve among the various points given by the data it should be remembered that zero tension corresponds to zero length so that the curve may be extended to pass through the origin.

The effect of the density of the material of the wire or string on the velocity of transverse waves may be strikingly shown by stretching side by side on the sonometer a steel wire and a piece of tough cord (fishing twine or gut) of about the same diameter (when stretched) as the wire. These are each pulled by means of weights attached to strings running over the pulleys until the pull per unit area of cross-section is the same for each. The vibrating lengths that will give the same note are then compared with the stretching forces per unit area of cross-section.

Determination of Pitch by Comparison with a Standard Tuning-Fork. The sonometer is frequently used for the comparison of notes of quite widely different pitch. It is desirable however to obtain a fork of nearly as possible the same pitch as the note which is to be determined. The pull on the sonometer string is adjusted until the two notes to be compared can be obtained on the bridge string. Since the tension of the string is kept constant the frequencies of the notes are inversely as the corresponding lengths. The accuracy of the comparison will depend greatly on whether a sufficiently great length of string is obtained for each note that the string is tuned to. The error in tuning and in measuring the length is large for short strings.

Comparison of Nearly Equal Notes by means of Beats. Suppose that we have two sources of sound which furnish us with notes of frequency 254 and 256 vibrations a second respectively. If the two trains of waves come to a place at which they agree (at a given instant), they will not agree again until 127 vibration periods of the one and 128 vibration periods of the other have elapsed, that is an interval of half a second. There will therefore be two beats a second. The number of beats a second gives the difference in the vibration numbers of the two notes. Such an experiment does not tell us however which source is giving the higher note. If the two sources are tuning-forks, for example, it will be possible to make the two notes the same by adding small pieces of soft wax to the prongs of one of the forks. The fork to which the wax has been added to produce equality of pitch is the fork which had the higher pitch when the beats were counted.

Experiment 8. Determination of the Frequency of an Alternating Current with the Aid of the Sonometer. The alternating current of the city lighting system (110 volt circuit) is sent through a 16-candle-power lamp in series with a solenoid of a large number of turns of No. 24 copper wire (wrapped around a soft iron core). A soft steel or soft iron wire is used on the sonometer. The tension of the wire is adjusted until the portion of the wire between the two wooden knife-edges vibrates when one of the poles of the solenoid is held near its middle. The note obtained may not be the lowest note that can be obtained in this way, but may be some harmonic. The tension being kept constant, the length of the wire between the two wooden edges should be doubled, trebled, etc., until the greatest length, for which there is resonance, has been found. The frequency of this note may be determined by comparison with a tuning-fork. Suppose it to be n. Then the frequency of the alternating current is $\frac{n}{2}$. In the above experiment the soft iron or steel wire is attracted by the solenoid both for the positive and negative directions of the alternating current. During a complete current period the value of the current rises for example from the value zero to a positive maximum, falls to zero, increases to a maximum for the negative direction, and falls back to zero again. The frequency of the alternating current is therefore only one half the frequency of the vibrating wire as obtained in this experiment.

Stroboscopic Method of Measuring Frequencies. A circular disk, provided with radial slits at regular intervals, is rotated by hand or by a small motor. A pin is attached by means of a piece of soft wax to the prong of a tuning-fork whose frequency is to be measured. The fork is mounted so that the pin may be seen through any of the slits as they come opposite, the pin being parallel to a slit.

The effect may be intensified by having the pin shown against a luminous background. The fork is set in vibration and the disk is rotated, the number of revolutions per minute being adjusted until the pin on the prong of the fork appears motionless when viewed through the revolving slit. When this is the case it is evident that the fork returns to the same place during the interval which it takes one slit to move forward a distance equal to the distance between two successive slits. For the most delicate adjustment it is necessary that the pin should be seen only at the middle of a swing, for then the velocity of the pin has its maximum value if we consider the swing from one extreme position to the other. The above adjustment does not tell us how many swings the fork has made during the interval between observations through two successive slits. It is therefore necessary to rotate the disk slowly at first and to gradually increase the speed of rotation until the fork, as seen through the slits, appears motionless for the first time. Then if the number of slits in the disk be known and if the number of its revolutions per second be known, the frequency of the fork may be calculated.

Illumination of Sound-Waves. The layer of condensed air which forms the crest of a sound-wave in air may be rendered visible by means of an arrangement devised by Toepler and improved by Wood. The sound waves are produced by the passage of an electric spark between two terminals of an induction coil. A spark between magnesium terminals is adjusted so that it follows the first spark about 0.00001 second later. Light from the magnesium spark therefore falls on the crest of the sound-wave after it has gone an inch or two from the place where it originated. An image of the magnesium spark is formed by means of an achromatic lens on the edge of a card near the objective of a telescope. The light from the magnesium spark therefore does not enter the telescope when the intervening air is undisturbed, but the crest of one of the sound-waves will (if in suitable position) refract some of the magnesium light past the edge of the card into the telescope. The crest of the sound-wave then appears like the edge of a soap bubble, when viewed by means of a bright light. A full account of the arrangement of apparatus for the illumination of such sound-waves will be found in Wood's *Physical Optics*, pp. 79-83. This method is of great importance in testing the optical homogeneity of an extended medium.

QUESTIONS AND EXAMPLES.

1. The value of Young's modulus for the material of a wire is 2×10^{12} dynes per square centimeter and the radius of the wire is 1 mm. What force will be required to lengthen it by one part in a thousand?

2. A locomotive blowing a whistle that gives a note containing 400 complete vibrations a second moves past and away from a station where an exactly similar whistle is blowing. An observer at the station counts five beats a second between the two notes. What is the speed of the train that is going away?

3. For a certain steel wire 0.5 mm. in diameter the value of Young's modulus is 2×10^{12} dynes per square centimeter, the breaking tension is 7000 kilograms per square centimeter. Compare the frequencies of the fundamental transverse and longitudinal notes when the wire is stretched nearly to its breaking tension.

4. Two gases have for the ratio of the specific heats the values 1.41 and 1.66 respectively, while the corresponding densities at a given temperature are in the ratio 4 to 1. What will be the relative velocities of sound in the two gases?

5. What would you infer as to the number of atoms to the molecule in the case of each of the gases in the preceding question?

II. DIFFRACTION AND INTERFERENCE EXPERIMENTS IN SOUND — ABSORPTION IN ROOMS.

Diffraction. The diffraction of sound-waves is more generally known than the diffraction of light-waves. Sound-waves of low frequency pass very easily around small obstacles, and the latter do not produce a pronounced sound-shadow in such a case. The shadow from a given obstacle becomes much more pronounced if the pitch of the note of the given source of sound be raised. Very high notes may be produced by the jangling of keys, etc. As a detector for sound-waves of very high pitch the sensitive flame discovered by Professor John LeConte is very frequently used. A mica vane suspended by a fine fibre may also be used. Near the borders of a sound-shadow the intensity of sound does not vary continuously, but in an intermittent way, in passing from the region of loud sound into the depths of the sound-shadow.

Interference. We have had a number of cases of interference and reinforcement in the experiments already outlined. In the Kundt's tube there is reinforcement of the direct wave by the reflected wave at the loops and interference between the two trains of waves at the nodes. The intermittent character of the sound in passing behind an obstacle is due to interference and reinforcement of waves diffracted from various portions of the original wave-front. The following experiment due to Quincke gives an excellent example of the interference of sound-waves.

Experiment 9. Quincke's Interference Apparatus. Koenig's Manometric Flame. A train of sound-waves (produced by a tuning-fork reinforced by a resonator at the mouth of the tube AD in figure 3) passes along the tube AD to D where it divides into two trains of waves passing along the paths DEG and DFG respectively. At G where the two trains of waves recombine there will be reinforcement of one train of waves by the other if the two paths from D to G are of equal length or differ by an integral number of wave-lengths. There will be interference and consequent silence at G if the two paths differ by any odd integral number of half wave-lengths. The reinforcement or the interference may be detected by means of lycopodium powder which has been scattered in the tube between G and H, or a rubber tube leading to a manometric flame may be attached at H. Instead of a tuning-fork at the mouth of the tube AB, a glass rod held at the middle with a cork which fits into the tube AB may be used. This will give

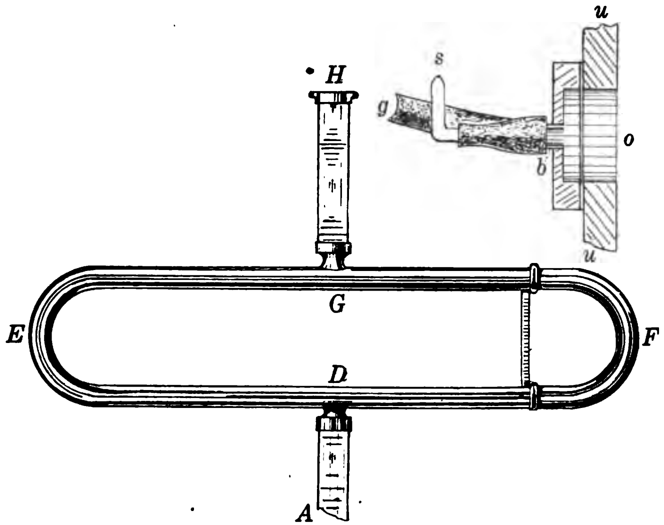


Fig. 3. Quincke's Interference Apparatus.

a note of great intensity when the rod is stroked with a cloth that has been soaked in alcohol.

Koenig's manometric flame is described in Watson (p. 415, fig. 263). As there described the flame is used to show the stationary waves in an organ pipe. In the present experiment a single rubber tube *g* is led from the Quincke tube GH to the chamber *o*. The latter is bounded by an elastic membrane by means of which the changes of pressure are transmitted to the burning gas of the manometric flame. When there are changes of pressure on the membrane, these cause the flame to vary in height periodically, the period being the same as that of the membrane. The character of these vibrations may be ascertained by viewing the reflection of the flame from the surface of a revolving mirror.

Determine the velocity of sound in a glass rod by Quincke's interference method. Examine the manometric flame used in connection with the tube by means of a revolving mirror and make a drawing of the flame picture. Report the harmonics of the fundamental note that may be present, and calculate their frequency.

Interference Experiments in Unconfined Air. Such experiments are often rendered difficult by disturbances from drafts and from reflections of sound from the floor and walls of the laboratory. If a pure note of very high pitch be used, however, many experiments (quite similar to the interference experiments usually performed in light) may be successfully performed with sound-waves. A form of sensitive flame used in the detection of sound-waves consists of a pin-hole burner (the orifice being a hole about 0.5 mm. diameter drilled in a piece of soapstone) in which the gas is allowed to burn above a piece of wire gauze supported a few centimeters above the orifice. The flow of the gas at the orifice is regulated by means of a pinchcock on the rubber connecting-tube until the flame is on the point of flaring. The flame is protected from drafts by a chimney. Sound waves of a sufficiently high pitch even though of small intensity will throw such a flame into violent movement if the waves strike the orifice.

Lord Rayleigh measured in the open air the wave-length of the sound from a high-pitched whistle. It was blown steadily at a point a few feet in front of a large wall. The sensitive flame was moved back and forth on a line drawn from it perpendicularly to the wall. In such an experiment the flame is least disturbed at the nodes (of which the wall is one) and most disturbed at the loops. The distance between any two consecutive nodes is half a wave-length as in the case of Kundt's experiment. If a tube leading to the ear is used (instead of the sensitive flame) the greatest effects are perceived at the nodes.

The effect of reinforcement and interference of sound-waves in the open air may be very strikingly shown by means of a zone plate. We may suppose a source of sound at the point S (see figure 4) which gives off sound-waves of length λ. A sheet of metal lies in a plane containing the point O and at right angles to the line SO. The point R is on the line SO produced, RO being equal to OS. Concentric spheres are described with radii $(SO + \frac{\lambda}{4})$, $(SO + \frac{\lambda}{2})$, $(SO + \frac{3\lambda}{4})$, $(SO + \lambda)$, etc. Call the lengths

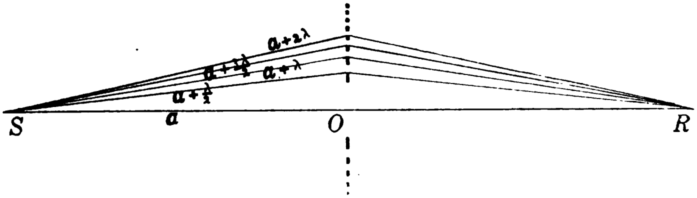


Fig. 4. To Illustrate Construction of Zone Plate.

of these radii SM_1, SM_2, SM_3, SM_4, etc., and call SO by the letter *a*. The central zone whose center is O and radius OM_1 is cut from the metal sheet and each alternate zone after that is cut out as indicated in the figure. The remaining zones are held together by means of two wooden strips which run across the metal strips at right angles to the zones.

If now a sensitive flame be placed at R while the given note is sounded at S, it will be found that the flame is much more strongly affected than when the zone plate is removed. If the zone plate be replaced and the sensitive flame be moved to one side of the axis SOR, a point is soon reached where the flame is hardly affected at all. The large effect at R when the zone plate is present is due to the resultant effect from the various zones that have been cut from the screen. The average distance from S to R by way of the zone M_1 M_2 is a half wave-length greater than is the average distance from S to R through the central zone. Thus the sound-waves that are cut off by the intermediate closed zone would (if it were open) interfere with and nearly destroy the waves that pass through the open central zone. Similarly it may be shown that the waves that pass through the open zone M_2 M_3 reinforce those which pass through the open central zone. The effect at R is therefore found to be much greater than when the zone plate is not present.

Young's experiment which shows interference bands from two similar sources in light may also be performed with sound-waves.

Conduction of Sound. In a homogeneous medium containing a source of continuous sound the intensity at any point is inversely as the square of the distance from the source. A tube of air therefore conducts the sound from one point to another much better than is the case when the sound-waves are allowed to spread through the whole medium surrounding the points. If the tube used for conveying the sound-waves is narrow, the amplitude of the vibrations is soon reduced by friction of the air against the walls of the tube, and much of the energy of vibration is quickly turned into heat. Speaking tubes an inch in diameter are therefore much more efficient in an office building than tubes a centimeter in diameter. Strings are also used as conductors of sound as in the case of the string telephone. If the string is loose part of the energy of longitudinal vibration is transformed into energy of transverse vibration. If thick rods are used to conduct sound part of the energy of longitudinal vibration is changed into energy of radial vibration. The velocity of the transmission of sound in a rod depends as we have seen on the elasticity and density of the material composing the rod, being proportional to the square root of Young's modulus and inversely as the square root of the density. Thus steel conducts sound four times as rapidly as lead. Very inelastic substances are very poor conductors of sound, as for example clay or decayed wood. If a medium be not homogeneous the conduction of sound through it is disturbed by reflections and refractions. Thus sound is conducted extremely well in a dense fog where the atmosphere is nearly homogeneous, while when the sun is shining there are many inequalities of density in the air. Sound-waves are then reflected or refracted at places where there is a change in density, so that they may be diverted from the desired point. In the testing of castings or of boiler plates the workman judges of the continuity of the material by the character of the sounds given out when different parts are struck in succession. Thus a fault in the boiler plate will reflect back a portion of the sound-waves.

Absorption of Sound in Rooms. The study of the absorption of sound by various materials is of great importance for the construction of halls and lecture rooms. It is necessary for good hearing in a room that the reverberation should not be too great or too little. In the case of symphonies for example it has been found (by a comparison of some of the best halls for symphony orchestras) that a period of reverberation of about three seconds is desirable. In the case of a piano in a music room a period of 1.1 seconds is best. For speaking, open air conditions where the reverberation is a minimum are the best. Different rooms may be compared by sounding in them successively the same organ-pipes with about the same intensity and noting the time taken in each case for the sound to disappear after the pipe has ceased to sound. As the sound decays very rapidly at the end it is possible to get very concordant measurements. The period of reverberation appears to depend very little on the shape of the room; but it depends on the volume, the area, and the materials of the walls. Professor Sabine found the following relation:

$$T = \frac{KV}{a}$$

where T is the time of the duration of the audible sound, K is a constant depending on the initial intensity, V is the volume of the room, and a is the total absorbing power of the walls and of the contained material. K is 0.164 for an initial intensity 10^6 times the minimum audible intensity. It does not vary with the pitch of the note. The absorbing power of the walls and contained materials however varies greatly with the pitch of the note. The absorption of felt for example is proportional to the thickness for a note of 64 vibrations a second; but for a note 6 octaves above this, the absorption is quite independent of the thickness of the material. Similar experiments have been made with chairs, cushions, wood sheathing, and even for an audience in a room. An audience absorbs practically all the sound that falls upon it for frequencies above 1024 vibrations a second, but shows very little absorption for low notes of frequency 64.

Students wishing to study this subject will find full accounts of the principal work in the subject in papers by Sabine in the *Engineering Record* and *American Architect,* 1899-1900, and in the *Contributions from the Jefferson Physical Laboratory,* 1905.

QUESTIONS

1. Why is sound conducted better in a dense fog than in ordinary fine weather conditions?

2. Why is it easier to conduct interference experiments in sound in the open air when notes of high frequency are used?

3. A wire is stretched under a tension of 10,000 grams per square centimeter. The value of Young's modulus for the material is 1.5×10^{12} dynes per square centimeter. Compare the frequencies of the fundamental longitudinal and transverse vibrations.

4. In question 3 what effect on the frequencies of the notes would the following changes have, the other quantities remaining constant during the change of any one:—(*a*) doubling the length of the wire, (*b*) trebling the stretching force per unit area of cross-section, (*c*) halving the area of cross-section of the wire while the stretching force remains the same?

5. Give instances of diffraction effects in the case of sound-waves, of interference effects. Describe an interference experiment in sound that has been performed in the open air.

6. Why do sound-waves in air at a distance from the source of sound not show polarization phenomena?

III. COMPOSITION OF VIBRATIONS.

Experiment 10. The Pendulum. A heavy pendulum bob is suspended by a long fine wire of steel or brass. The bob has a hole drilled through the axis of figure, and in this opening the tube of a funnel containing dry fine sand is placed. The path of the pendulum bob is then recorded by means of the fine stream of sand falling on the table below. The student can now determine by actual experiment what types of vibration are possible as a resultant of two equiperiodic simple pendulum vibrations at right angles. After a study of the subject he should be able to predict the curves that would result in the following cases. For the purposes of definition O is taken as the point (on the table) that lies directly under the center of gravity of the bob in its position of rest. Lines NOS and EOW are drawn on the table at right angles to each other. What then will be the resultant vibration when (1) the bob being over O an impulse is given it in the direction OS and at the same time twice as great an impulse is given it in the direction OW; (2) the bob being held over a point on the table 8 inches north of O, it is started off by means of an impulse at right angles to a vertical plane through NOS and sufficient to carry the bob 8 inches in an easterly direction; (3) the bob is started from a point 5 inches north of O by giving it an impulse sufficient to send it 3 inches farther in a northerly direction and by a simultaneous impulse sufficient to send it 8 inches in a westerly direction?

The resultants of vibrations of unequal periods may be shown and recorded by a pendulum like the one indicated in figure 5. The pendulum bob is supported by three strings so that for vibrations in the plane ACB the length of the pendulum is CE, for vibrations in the plane at right angles, the length of the pendulum is ED. The two periods of vibration are proportional to $\sqrt{CE}$ and $\sqrt{DE}$ respectively.

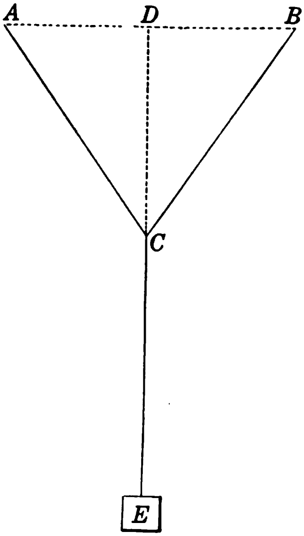


Fig. 5 Pendulum for Composition of Vibrations in Planes at Right Angles.

The student is asked to obtain figures showing the resultants obtained by combining vibrations at right angles for the cases where the two frequencies have the ratios $\frac{2}{1}$ and $\frac{3}{2}$ respectively. The forms of the figures in each case will of course change somewhat according to the relative amplitudes and phases of the two components.

Experiment 11. Combination of Vibrations at Right Angles by Means of the Harmonograph. The vibrations may be recorded in much more permanent form by means of a glass pen as shown in figure 6. The two pendulums that vibrate at right angles are supported at A and B (the supports not being shown in the figure). The periods may be varied by adjusting the positions of the masses W_1 and W_2. Paper is attached to the cylindrical table T. The pen P is made by drawing a glass tube down to a very fine tip. The pressure of the pin may be nicely adjusted by means of the weight F. The records obtained by this instrument should be preserved in the note book with a statement of the periods of vibration of the pendulums in each case.

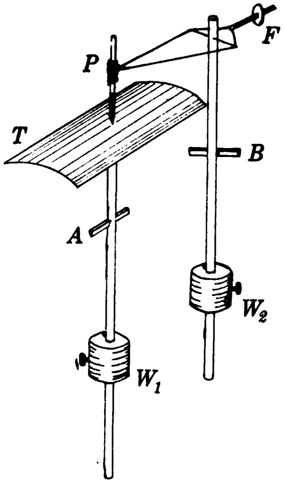


Fig. 6. The Harmonograph.

Experiment 12. Simple Wave Form. Combinations of Waves. With the aid of a pendulum that has a funnel attached to the bob as in experiment 10, a simple wave form may be obtained as follows: the pendulum is started in vibration in a given plane by first displacing the bob a little by means of an attached thread and then igniting the thread. The pendulum will then vibrate for some time in one plane. A large sheet of cardboard is drawn across (with as uniform a speed as possible) the table in a direction at right angles to this plane. Indicate this direction by a line drawn in a vertical plane through the position of the center of gravity of the bob when at rest. This line will bisect the wave form which cuts off from it equal distances l if the cardboard has been drawn along the table at uniform speed. The crests of the waves will be distant a from this line where $2a$ is the amplitude of the vibration of the bob. Compare the wave form obtained with that given by the equation

$$y = a \sin\left(\frac{x}{l} \cdot \pi\right),$$

where x denotes a distance along the bisecting line measured from one of intersections as origin and y denotes a distance measured at right angles to the direction of x. The point x, y is any point on the sine curve.

By decreasing the length of the pendulum to one fourth its former value a wave form may be obtained which indicates twice the frequency of the former vibration. This form can be represented by an exactly similar equation

$$y = a' \sin\left(\frac{x}{l'} \cdot \pi\right)$$

Plot both of the above equations on coordinate paper representing a for example by a distance equal to 2 centimeters and l by a distance equal to 3 centimeters, using the same origin and axes for both curves. Suppose a' to be equal to one fourth of a and l' equal to $\frac{l}{2}$.

Obtain a third curve which represents the resultant of the two wave forms by adding the ordinates of the first two curves. This may be taken as an example of a wave form obtained when in addition to a fundamental vibration the first harmonic is also present.

LIGHT

I. DIFFRACTION AND INTERFERENCE OF LIGHT-WAVES

General Properties of Ether-Waves. While sound-waves in liquids and gases are longitudinal waves, light-waves are shown by the phenomena of polarized light to be transverse waves, that is the vibrations are in planes at right angles to the direction of propagation of the light. Sound-waves are not propagated in empty space. Light-waves traverse interplanetary space with a velocity of 300,000 kilometers a second. Eclipse phenomena indicate that the velocity of light in interplanetary space is the same for light of all colors. This is not the case in air or other forms of matter. In passing through matter the vibrations constituting red light have in general a greater velocity than the more rapid vibrations which constitute violet light. Red light passes through water with about three fourths of the velocity that it has in empty space. The velocity of violet light in water is about one percent less than that of red.

Monochromatic light consists of ether vibrations which have all the same frequency. The light given by a Bunsen flame which contains some vapor of a sodium salt consists almost entirely of vibrations whose frequencies lie between 509.4 trillions a second and 508.9 trillions a second. Therefore, although it is not absolutely monochromatic, it is sufficiently so for most physical measurements. This frequency is too great to measure directly by experiment, but it may be obtained through measurements of velocity and wave-length. The velocity of sodium light in air differs very little from its velocity in a vacuum and is equal therefore to 3×10^{10} centimeters per second. The wave-length may be measured by interference methods, such as Young's experiment, which may also be used with very short sound-waves. This method gives for the mean wave-length of sodium light about 0.000059 centimeter. The corresponding value of the frequency is then given by

$$n = \frac{v}{\lambda} = \frac{3 \times 10^{10}}{5.9 \times 10^{-5}} = 5.1 \times 10^{14}$$

vibrations a second approximately.

The colors red, yellow, etc., are not exact scientific terms but each include light-vibrations of a large range of frequency. The following table gives an approximate mean value for the frequency of the light-vibrations of different parts of the sun's spectrum, together with the corresponding values of the wave-length in air.

Red	$\lambda = 0.000068$	centimeter	$n = 440$	trillion	a second
Orange .	0.0000615	"	490	"	"
Yellow .	0.000056	"	540	"	"
Green ..	0.000051	"	590	"	"
Blue	0.000047	"	640	"	"
Indigo ..	0.000044	"	680	"	"
Violet ..	0.000041	"	730	"	"

The human eye has a range of only a little more than an octave, from about 360 trillion a second (the extreme red) to 790 trillion a second (the extreme violet), the respective wave-lengths being about 0.000083 cm. and 0.000038 cm. Vibrations of somewhat greater frequency (therefore in the ultra-violet) than the violet are very active in producing chemical changes in photographic plates. Vibrations of as great frequency as 2500 trillion a second (about two octaves above the violet) have been detected by photographing the light in an atmosphere of hydrogen. Air absorbs vibrations of such high frequencies very strongly. The infra-red rays (with frequencies less than the red) are present in great intensity in the sun's spectrum. They may be detected by means of thermal junctions connected to a galvanometer. Long ether-waves of frequencies of about a million a second and of wave-length about 300 meters are used frequently for long distance wireless telegraphy. They may be detected by means of coherers in connection with a battery and a galvanometer. Such waves bend around large obstacles such as mountains. This is an instance of diffraction of ether-waves.

Experiment 13. Diffraction Patterns and Interference Bands. A piece of tin foil is smoothed out on a plate of glass. A sheet of letter paper is then slipped between the foil and the glass, and with a fine needle an opening 0.1 to 0.4 millimeter in diameter is made in the foil. The foil is now removed from the paper and placed in front of the eye, which is turned toward an arc light several feet away. If the opening is circular, colored rings will be seen around a white center. The smaller the opening the larger the diameters of the rings. Can the rings be equally well obtained when a large luminous gas flame is viewed through the opening? Try also the filament of an incandescent lamp.

Having made a very small opening (that gives well-defined colored rings, when a small luminous source is used), mount the tin foil on a piece of cardboard so that the diffraction pattern from the opening in the tin foil may be viewed with a micrometer eye-piece. The cardboard is clamped in a standard on the optical bench. A tube to hold the eye-piece is mounted on the same bench, the axis of the tube being in line with the luminous source and the circular opening. When the eye-piece has been replaced, the diffraction pattern now much magnified will probably be seen. The eye-piece is focussed on the cross-hairs. In measuring the mean distance between two consecutive rings of red light, for example, the initial and final readings of the micrometer screw should be taken after rotations of the screw in the same direction. This enables one to avoid the errors that may otherwise be caused by backlash of the screw. The student may now compare the mean distance between two consecutive red rings with the mean distance between two consecutive blue rings in the same neighborhood. These measurements enable one to calculate the relative mean wave-lengths of red and blue light in the spectrum of the arc. The measurements and calculations should be reported in the note book. The student is also asked to obtain and describe with drawings the patterns given by the following arrangements. The dimensions are only intended to indicate the order of magnitude of the quantities involved and need not be exactly followed: (1) two circular openings one or two tenths of a millimeter in diameter and distant about one half a millimeter from each other, (2) like (1) but with a third opening equidistant from the other two, (3) four openings at the corners of a square whose side is 0.5 millimeter, (4) a long straight slit made in a piece of tin foil that has been attached to glass with a thin layer of paraffin, (5) two parallel slits similar to that in (4) about 0.5 millimeter apart, (6) the edge of a large rectangle cut from a piece of tin foil, (7) a fine needle mounted in front of a circular opening about two millimeters in diameter.

Experiment 14. Measurement of Wave-Length by means of Young's Interference Bands. Two small openings about a third of a millimeter from center to center are made in tin foil as described in experiment 13. It is well to make several trials testing the preparations by holding the openings in front of the eye, when the latter is directed toward the arc light. Select the preparation that gives the most distinct interference bands and mount it for measurement of the bands as described in experiment 13. To obtain an approximate mean value of the wave-length for the visible spectrum of the arc, the mean distance between the centers of two dark bands may be used. These bands are of course only found near the center of the system of bands. It is necessary also to measure the distance between the two openings in the tin foil, and again the distance from the openings to the plane of the cross-hairs where the particular bands (that we are concerned with) lie. The latter distance is so large that it can be measured with sufficient accuracy by means of a meter rod. To obtain the distance between the openings a convergent lens is mounted between the openings in the tin foil and the cross-hairs. If for example the focal length of the lens is 15 cm., the best arrangement will be to mount the lens about 30 cm. from the tin foil and to place the cross-hairs about 30 cm. on the opposite side of the lens from the foil. The openings are illuminated with the arc or any convenient source. When an image of the two openings has been obtained the distance between the two openings as given in the image is measured by means of the micrometer screw and attached cross-hair. The corresponding object distance may then be obtained from the relation

$$\frac{\text{Distance between two openings in foil}}{\text{Corresponding distance in image}} = \frac{\text{Distance of object from lens}}{\text{Distance of image from lens}}$$

We have now all the data necessary for calculating the wave-length from the relation

$$\frac{\text{Wave-length for light of given color}}{\begin{array}{c}\text{Distance between consecutive bands}\\ \text{of given color}\end{array}} = \frac{\text{Distance between two openings}}{\begin{array}{c}\text{Distance from openings to}\\ \text{cross-hairs}\end{array}}$$

If the bands are very distinct it may be possible for the student to determine the wave-length for some given color. If he is unable to obtain them with sufficient distinctness for this purpose, it will be sufficient to measure the distance between two consecutive dark bands near the center of the system and so obtain a mean value of wave-length for the arc spectrum.

Theory of Young's Experiment. The above expression for the wave-length may be very simply derived from the fact that when the distances from the two openings to a given point in the plane of the cross-hairs differ by one wave-length or any whole number of wave-lengths, the vibrations from the two openings will reinforce each other if we suppose them to start alike at the two openings. If $QP_1 - QP_2 = n\lambda$ (see figure 7), where $n\lambda$ represents any whole number of wave-lengths, Q will be on a bright band. O is a point equidistant in the plane of the cross-hairs equidistant from P_1 and P_2. Then O is the central band which is bright for all colors. C is a point halfway between the two openings P_1 and P_2. Now in the two triangles P_1RP_2 and COQ, we have OC perpendicular to P_1P_2, and CQ perpendicular to RP_2. Therefore the angle P_1P_2R is equal to the angle OCQ. The angles P_1RP_2

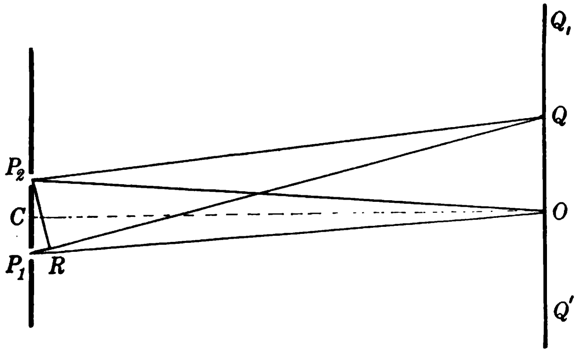


Fig. 7. Light-Waves from P_1 and P_2 Form Interference Bands on a Plane Perpendicular to the Paper Through OQ.

and COQ are each right angles. Therefore the two triangles are equiangular and similar.

Therefore

$$\frac{P_1R}{P_1P_2} = \frac{OQ}{OC} \text{ or } n\lambda = \frac{P_1P_2}{OC} \cdot OQ$$

If the next bright band be at Q_1 we have

$$(n + 1)\lambda = \frac{P_1P_2}{OC} \cdot OQ_1.$$

Subtracting one equation from the other we get

$$\lambda = \frac{P_1P_2}{OC} \cdot QQ_1.$$

Therefore the wave-length is equal to the distance between two consecutive bright bands multiplied by the distance between the openings and divided by the distance from the openings to the cross-hairs. Instead of the distance between two consecutive bright bands we may use that between two consecutive dark bands since the bright and dark bands for monochromatic light are equally spaced.

Experiment 15. Measurement of Wave-Length of Sodium Light by means of Newton's Rings. A plano-convex lens is placed with the convex side down on a plane surface of black glass. The radius of curvature of the convex surface may be conveniently 400 to 800 centimeters. This makes the air-gap between the convex and plane surfaces very small (even in comparison with the wave-length of the light used) in the neighborhood of the point of contact. The thickness of the air-gap increases for some distance as the square of the distance from the point of contact. The source of light is a Bunsen flame whose edge is placed against a piece of asbestos with some common salt on it. A lens may be placed at a distance from the flame equal to its focal length in order to make the light that comes from any point on the Bunsen flame a beam of parallel rays. This beam falls on a plane glass surface (that makes an angle of 45° with the vertical) and it is reflected so as to fall perpendicularly on the plane surface of the plano-convex lens. Some of the light is reflected upward again from the concave surface of the air-gap, some more passes through the air-gap twice, being reflected upwards from the surface of the black glass. The light which is reflected in these two ways and passes up through the 45° plate, is brought to a focus in the plane of the cross-hairs of a microscope provided with a micrometer eye-piece. The interference bands obtained in this way will evidently be rings concentric with the nearest point of approach of the convex and plane surfaces. The distance between two consecutive bright bands of the same color depends on the wave-length of the light in air and on the radius of curvature of the convex surface. Watson (pp. 544-545) assumes that the convex surface touches the plane surface. As this may not be the case on account of small grains of dust, serious error might be made in calculating from the formula given by Watson for the radius of a given bright ring. If we measure instead the differences of the radii of rings we may calculate the wave-length independently of whether the two surfaces touch or not as long as the distance between them remains constant during the measurement of the radii of the rings.

In figure 8 the notation is similar to that in Watson except that a second ring, (the $(n+1)$st ring, is added; and at the nearest point between the two surfaces the distance is supposed to be OO′. Then for the difference in the thicknesses of the air-gaps at P and P′ we have

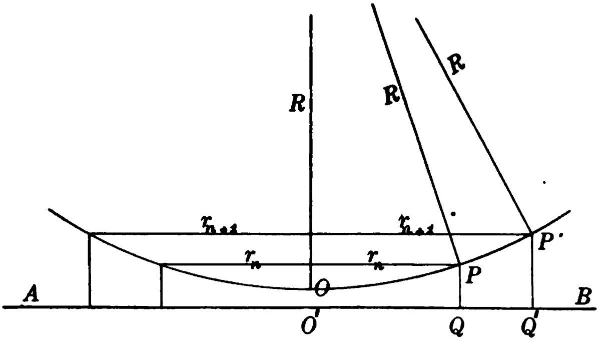


Fig. 8. Illustrating Newton's Rings.

$$P'Q' - PQ = \frac{\lambda}{2}$$

$$\text{But } PQ = OO' + \frac{r_n^2}{2R} \text{ and } P'Q' = OO' + \frac{(r_n + 1)^2}{2R}$$

$$\text{Therefore } \lambda = \frac{(r_n + 1)^2 - r_n^2}{R}.$$

The distance between the squares of the radii of two bright bands is a constant, and if we consider only the brightest portions of the rings we have a series of concentric circles such that the area between any two consecutive circles is a constant. This enables us to understand at a glance why the rings crowd closer together as the distance from the center of the rings increases. For accuracy of measurement it is therefore better to use rings near the center of the system. If we take for example the first bright ring and the fourth one beyond it we have

$$4\lambda = \frac{r_5^2 - r_1^2}{R} = \frac{(r_5 + r_1)(r_5 - r_1)}{R}$$

The measurements with the micrometer screw may be resolved into a measurement of the sum of the radii of two circles, that is from the circumference of one through the center to the opposite circumference of the other, and a measurement of the distance between the two circumferences. The latter measurement involving a smaller distance needs to be taken with more care. The cross-hairs of the travelling microscope should always be perpendicular to the direction of its motion. The initial and final readings for any given measurement should always be taken after rotation of the screw in a given direction, in order to avoid errors due to backlash. The value of the radius of curvature of the convex surface (as determined by one of the instructors from Newton's rings measurements) is given on a label attached to the lens.

Determine the wave-lengths of sodium light in air and water respectively.

While the measurement of the rings is very easy in the case of air, it is more difficult in the case of water on account of insufficient intensity of the light reflected up through the system. This part of the experiment may be omitted if there is not time to finish it within the period.

If the wave-length of sodium light is found to be λ_w in water, we have for the refractive index of water, $\frac{\lambda_a}{\lambda_w}$, for sodium light.

QUESTIONS.

1. In Young's experiment how far apart would the two sources of light have to be to give bright bands a millimeter apart in Thallium green on a screen a meter away from the sources? The wave-length in air for Thallium green is 0.00053 mm.
2. How can you compare the velocities of sodium light in air and in water by an interference method?
3. Explain the formation of a white center in Newton's rings. How could the refractive indices of the glass of the lens, of the glass with the plane surface, and of the fluid between the two be chosen so that there would always be a white center when the lens is in contact with the plane glass?

II. MEASUREMENT OF REFRACTIVE INDEX.

General Relations. We have seen that the refractive index of a substance can be determined for light of a given frequency by comparing the wave-lengths of the light in the given substance and in air respectively. We have

$$\frac{n\,\lambda_a}{n\,\lambda_s} = \frac{v_a}{v_s} = \mu,$$

where n denotes the frequency of the light vibrations, λ_a and λ_s the wave-lengths in air and the given substance respectively, and v_a and v_s the corresponding velocities.

The method where wave-lengths are compared is used in some refined measurements of refractive index, as for example in the measurement of the refractive index of a gas at a given temperature.

Refractive index is measured more commonly by a comparison of the velocities of light in air and in the given substance respectively. Let OO′ (fig. 9) represent the plane boundary between air and a given homogeneous substance, and let II′ represent a beam of parallel light in air, which meets the boundary in OO′ and proceeds in the substance as the beam RR′. MN is the normal to the boundary at O′. OP represents the wave-front at a given instant in air, O′Q represents the wave-front at a later instant in the given substance. The light travels the distance O′P in air in the same time that it travels the distance OQ in the substance. Therefore

$$\mu = \frac{v_a}{v_s} = \frac{O'P}{OQ} = \frac{\dfrac{O'P}{OO'}}{\dfrac{OQ}{OO'}} = \frac{\cos \angle PO'O}{\cos \angle QOO'}.$$

But cos ∠ PO′O = sin ∠ PO′M = sin *i*, where the angle PO′M is the angle of incidence of the light. Also cos ∠ QOO′ = sin ∠ NO′R′ = sin *r*, where NO′R′ is the angle of refraction. Therefore

$$\mu = \frac{v_a}{v_s} = \frac{\sin i}{\sin r}.$$

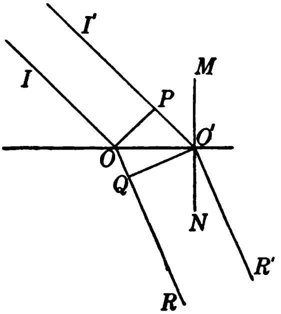


Fig. 9. Refraction at a Plane Surface.

Experiment 16. Preliminary Measurements of Refractive Index and Critical Angle. The angles of incidence and refraction may be approximately measured in the case of liquids by means of the following simple form of apparatus (see figure 10). A vessel that has a vertical side of thin glass is used. The opposite faces of the glass front should be plane and parallel to each other. A small strip of mirror is mounted in the vessel so that the mirror can be turned round a vertical axis. The vessel is placed on a large horizontal table on which the measurements are to be made. A white string one or two meters long is stretched along the surface of the table in a plane with the axis of the mirror. The vessel is then turned until the image of the string in the glass front of the vessel is in line with the string itself. This means that the front of the vessel is perpendicular to the string. The mirror is turned so that it makes an angle of ten or fifteen degrees with the glass front. This is equal to the angle SMN in figure 10, where MN is the normal to the mirror. The liquid to be tested is poured into the vessel until the surface comes a centimeter or two above the surface of the table. A ray of light (from the point S on the string) is reflected from the mirror at its middle point M, and passes along the paths MOP or MOQ according as it strikes the mirror above or below the surface of the liquid. If we can neglect the thickness of the glass, the index of refraction of the liquid μ is equal to $\frac{\sin \angle S'OQ}{\sin \angle S'OP}$, because S′OP is equal to the angle between MO and the normal at O. The distances OS′, S′P, and S′Q can be measured with a meter rod, the positions of P and Q being determined by sighting the string in the mirror above and below the surface of the liquid respectively. If the mirror M lies near the glass front and the distance MS is made large (1 to 2 meters) S may be taken as coincident with S′.

The critical angle is the value that S′OP assumes when S′OQ is 90°.

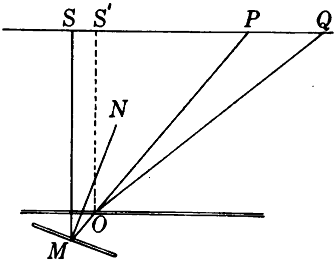


Fig. 10. Arrangement for Approximate Measurement of Refractive Indices of Liquids.

In making this measurement it will be found convenient to use an incandescent filament or lighted candle at S. The mirror is turned until the eye catches the rays OQ as they graze the glass front of the vessel. It will be found that the critical angle is different for light of different colors. This form of apparatus is not however well adapted for accurate measurements such as would show the dependence of refractive index on frequency.

If we denote by c the angle that S′OP becomes when S′OQ becomes a right angle, then the refractive index

$$\mu = \frac{\sin 90^\circ}{\sin c} = \frac{1}{\sin c} \text{ and } c = \sin^{-1}\frac{1}{\mu}$$

Determine approximately the refractive indices and critical angles (mean value for white light) of water and kerosene.

Experiment 17. Determination of Refractive Index by Measurement of Apparent Depth. In figure 11, A represents a point in the medium whose refractive index is to be determined. AO and AO′, representing rays coming from A, will be bent at the plane boundary (between the given medium and air) to the directions OP and O′P′, as if they came from a point A′. Then if MON be the normal to the plane surface through O, the refractive index is

$$\mu = \frac{\sin \angle \mathrm{PON}}{\sin \angle \mathrm{MOA}} = \frac{\sin \angle \mathrm{MOA'}}{\sin \angle \mathrm{OAB}} = \frac{\dfrac{\mathrm{OB}}{\mathrm{OA'}}}{\dfrac{\mathrm{OB}}{\mathrm{OA}}} = \frac{\mathrm{OA}}{\mathrm{OA'}}$$

In the limit as the cone of rays is made very narrow, OA becomes the distance from A to the plane surface and OA′ becomes the apparent depth of A below the surface, when A is observed by an eye on the upper side of the boundary in the air. Therefore the refractive index is

$$\frac{\text{Real Depth of A}}{\text{Apparent Depth of A}}.$$

As an example of the method we may find the refractive index of the glass in a specimen of plate glass. A may be the intersection of two very fine scratches on the lower side of the plate and B a point similarly marked on the upper surface of the plate. It is now mounted on the table of a travelling microscope which has been focussed on the top of the table. The position of the microscope index is read on the scale, fractions of a division being obtained by means of the micrometer screw. By turning the head of the micrometer screw the microscope is moved vertically up until the image of the point B is in the plane of the cross-hairs. This may be ascertained by moving the eye from side to side and noticing whether there is any relative motion of B and the cross-hairs. The difference between the initial and final readings for the position of the microscope gives the thickness of the glass plate. The microscope is now moved down by turning the head of the micrometer screw until an image of the point A is obtained. The corresponding shift of the microscope gives the apparent thickness of the glass plate.

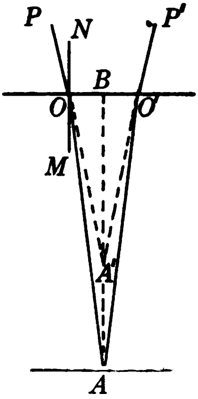


Fig. 11.

The refractive index of a liquid may be determined in a similar manner. The microscope is moved until the image of a point at the bottom of the empty vessel has been brought into the plane of the cross-hairs.

A convenient amount of the liquid is used and a little chalk dust scattered on the surface to make the image of the latter visible in the plane of the cross-hairs. The measurement is completed in the same way as before.

Determine the refractive indices (mean value for white light) of plate glass and water.

Experiment 18. Determination of Refractive Index by Measurement of Apparent Displacement through a Plane-Parallel Plate. Figure 12 indicates the displacement of a ray I, which after passing through the plate takes the parallel direction I′, *d* millimeters away. We see from the figure that

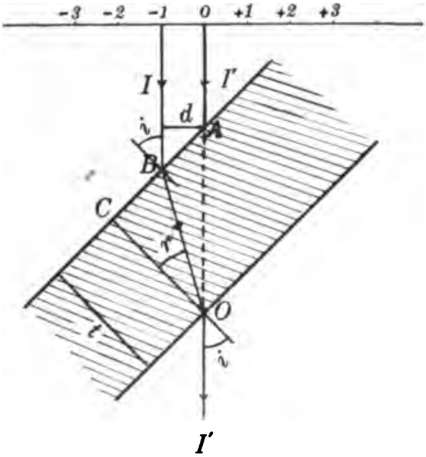


Fig. 12. Apparent Displacement Through a Plane-Parallel Plate.

$$d = AB \cos i = (AC - CB) \cos i = t\,(\tan i - \tan r) \cos i.$$

This gives $\tan r = \tan i - \dfrac{d}{t \cos i}$.

All the quantities on the right-hand side of the equation may be approximately measured by mounting a piece of plate glass over the center of a rotating table, there being a fixed angular scale concentric with the axis of the table. A millimeter scale is viewed (by means of a fixed telescope) across the edge of the glass plate, so that the scale is seen partly undisplaced (above the edge) and partly displaced by the glass plate (as seen below the edge). The millimeter scale should be perpendicular to the axis of the telescope. When the glass plate is parallel to the scale the divisions of the latter as seen below the edge will be in line with those seen above the edge. This may be called the zero position of the rotating table. It is read by means of an attached index. The table is then turned until a displacement of the scale amounting to one millimeter is obtained. Call the angle through which the table has been turned i_1. This is equal to the angle of incidence on the plate of the light that comes to the cross-hairs of the telescope from the scale. Let the corresponding angle of refraction in the plate be r_1. Then $\tan r_1 = \tan i_1 - \frac{1}{t \cos i_1}$. Hence $\sin r_1$ may be found from the tables, and $\mu = \frac{\sin i_1}{\sin r_1}$ may be calculated.

Refraction through a Thin Lens. The lens indicated in figure 13 is supposed to be of very small dimensions as compared with the distance of the object from the lens. Let this distance be u. Then the wave-front of the entering light just before it touches the glass is GCK. The wave-front just after it leaves the glass is HDL, the radii of these two arcs being u and v, the object and image distances respectively. Let the radii of curvature of the lens surfaces ACB and ADB be r_1 and r_2 respectively. Then AO is very nearly a mean proportional between CO and the

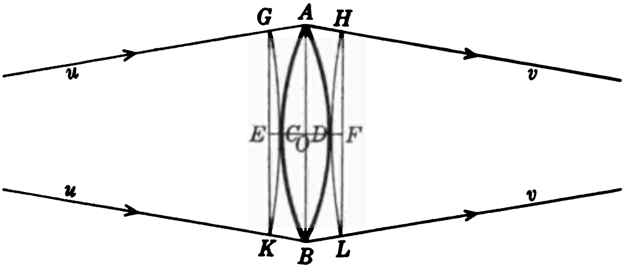


Fig. 13. Refraction Through a Thin Lens.

diameter (of the sphere) $2r_1$. Therefore

$$CO = \frac{\overline{AO}^2}{r_1}.$$

$$\text{Similarly } DO = \frac{AO^2}{r_2},\ EC = \frac{\overline{OA}^2}{u},\ DF = \frac{\overline{AO}^2}{v},$$

since AO, GE and HF are very nearly equal. Now the light takes the same time to travel GAH in air that it does to travel the distance CD in glass. Therefore the refractive index $\mu = \frac{GAH}{CD} = \frac{EF}{CD}$ since GH is equal to EF. Substituting for CO its value $\frac{\overline{AO}^2}{r_1}$, etc., we obtain

$$CD = \frac{\overline{AO}^2}{r_1} + \frac{\overline{AO}^2}{r_2}$$

$$\text{and } EF = \frac{\overline{AO}^2}{u} + \frac{\overline{AO}^2}{r_1} + \frac{\overline{AO}^2}{r_2} + \frac{\overline{AO}^2}{v}$$

Therefore the refractive index

$$\mu = \frac{EF}{CD} = \frac{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{u} + \frac{1}{v}}{\frac{1}{r_1} + \frac{1}{r_2}}.$$

Therefore

$$(\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{1}{u} + \frac{1}{v}$$

If the object distance u is infinite, the entering light is parallel and the image distance v becomes the focal length f. Thus

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{1}{u} + \frac{1}{v}.$$

The refractive index for the glass of a thin lens may therefore be determined by the measurement of its focal length and the radii of curvature of its two surfaces.

Refraction through a Prism at the Angle of Minimum Deviation. Measurements of the refractive index of a transparent substance can be made with great precision if an accurately made prism of the material is available. A section of a prism made at right angles to the refracting faces is indicated in figure 14. A ray traversing the prism between two faces which make an angle θ with each other is bent through the angle $\alpha_1 - \beta_1$ at the boundary where it enters, and is bent by the amount $\alpha_2 - \beta_2$ at the face of the prism which it leaves. The total deviation δ produced by the refracting angle θ is seen from the geometry of the figure to be $(\alpha_1 - \beta_1) + (\alpha_2 - \beta_2)$. Now it may be shown that the deviation δ is least when $\alpha_1 = \alpha_2$ and when therefore $\beta_1 = \beta_2$, that is when the total deviation is divided up equally at the two faces of the prism. In this case we have $\delta = 2(\alpha - \beta)$. The refracting angle θ is always equal to $\beta_1 + \beta_2$ and therefore in the position of minimum deviation $\theta = 2\beta$. But the refractive index is equal to

$$\mu = \frac{\sin \alpha}{\sin \beta} = \frac{\sin\left(\frac{\delta}{2} + \beta\right)}{\sin \frac{\theta}{2}} = \frac{\sin \frac{1}{2}(\theta + \delta)}{\sin \frac{1}{2}\theta},$$

where δ in the formula is the angle of minimum deviation.

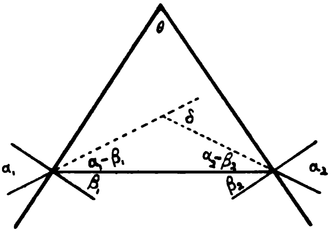


Fig. 14. Refraction Through a Prism.

The angle between two faces of a well-made prism can be measured with great accuracy by means of a spectrometer. The position of minimum deviation may easily be obtained with sufficient accuracy, because in the neighborhood of this position a considerable rotation of the prism produces little change in the deviation of the ray. It is possible with a spectrometer and prism to determine the variation of the refractive index of the material of the prism with the frequency of the light used. This problem will be deferred until we take up the study of the spectrometer.

Experiment 19. Measurement of Critical Angle and Refractive Index with Pulfrich's Refractometer. One end of a cylindrical tube is ground on a piece of plate glass and then mounted on the face of a right-angle prism as indicated in figure 15. A little vaseline is rubbed on the ground edge before the liquid is transferred to the vessel with a pipette. If the refractive index of the liquid is less than that of the material of the prism some of the light (from a sodium burner) that nearly grazes along the liquid-glass boundary will enter the prism. Suppose the angle of incidence is α_1 and the angle of refraction β_1. Let the corresponding angles at the other face that bounds the refracting angle θ be β_2 and α_2 in glass and air respectively. This supposes that the ray strikes this latter face at less than the critical angle for the glass-air boundary. Let N be the refractive index of the liquid and n the refractive index of the glass of the prism. Then $\frac{N}{n} = \frac{\sin \beta_1}{\sin \alpha_1}$ or when $\alpha_1 = 90°$, $N = n \sin \beta_1$. From the geometry of the figure we see that $\beta_1 + \beta_2 = \theta$. Again

$$n = \frac{\sin \alpha_2}{\sin \beta_2}.$$

Therefore

$$N = n \sin \beta_1 = n \sin (\theta - \beta_2) = n \sin \theta \cos \beta_2 - n \cos \theta \sin \beta_2.$$

But $\sin \beta_2 = \frac{\sin \alpha_2}{n}$. Therefore $\cos \beta_2 = \sqrt{\frac{n^2 - \sin^2 \alpha_2}{n^2}}$.

$$\text{Thus } N = n \left\{ \sin \theta \sqrt{\frac{n^2 - \sin^2 \alpha_2}{n^2}} - \cos \theta \frac{\sin \alpha_2}{n} \right\}$$

$$= \sin \theta \sqrt{n^2 - \sin^2 \alpha_2} - \cos \theta \sin \alpha_2.$$

Hence the refractive index N of the liquid may be calculated if n the refractive index of the glass is known, and if the angles θ and α_2 have been measured. The angle θ may be measured with a spectrometer. The angle α_2 is determined by the boundary between the bright and dark regions as indicated in the figure. It is measured by means of an angular scale and telescope attached to the apparatus. The student will find farther details as to the setting up of

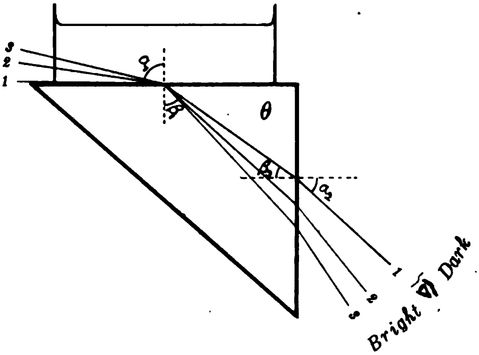


Fig. 15. Pulfrich's Refractometer.

the apparatus in Wiedemann and Ebert's *Physikalisches Praktikum* or in Minor's *Physical Measurements.*

Determine the refractive indices of water, alcohol, and coal oil with this apparatus.

QUESTIONS.

1. Light is incident on the boundary between a piece of plane glass (whose mean index of refraction is 1.5) and carbon bisulphide. If the light comes to the glass through carbon bisulphide whose corresponding index is 1.67, calculate approximately the greatest angle of incidence at which light can pass into the glass.

2. What will be the focal length of a thin double convex lens, the radii of curvature of the two surfaces being 15 cm. and 20 cm. respectively, for the following colors for which the index of refraction of the glass is given:

$$\mu_D = 1.582,\ \mu_F = 1.600,\ \mu_H = 1.631?$$

3. What would be the angles of minimum deviation produced by a prism made of the glass in question 2 for each of the above colors through a refracting angle of 60°?

4. A plano-convex lens is to be made of the glass in question 2 and is to have a focal length of 40 centimeters for sodium light. What must be the radius of curvature of the convex surface? What will be the focal length for the other two colors mentioned in question 2?

III. STUDY OF CONSTITUENT PARTS OF OPTICAL INSTRUMENTS.

Experiment 20. Measurement of Radius of Curvature of Spherical Surfaces with the Spherometer. (*a*) **The Ring-Spherometer.** Let SOP (fig. 16) be the surface whose radius of curvature is to be determined. Let SP be the diameter of the ring on which the spherical surface is placed. If the axis of the micrometer screw is in line with the center of the ring, the "bulge" of the sphere below the plane STP, that is the distance OT in the figure, may be measured by the micrometer screw. To bring the point of the screw to T it is necessary after the reading for O has been taken to remove the spherical surface and put a piece of glass (with a plane lower surface) across the ring. By the geometry of the figure we have

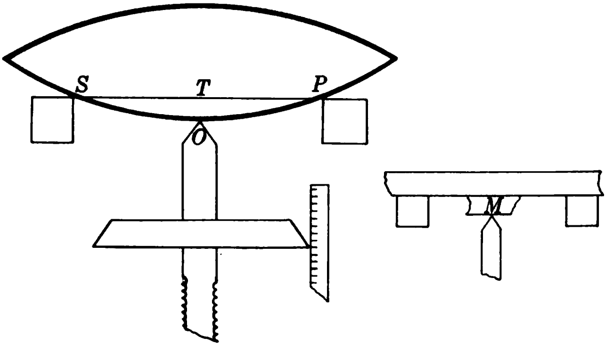


Fig. 16. The Ring-Spherometer.

$$\overline{ST}^2 = \overline{OT}.\ (2\ R - \overline{OT})$$

where R is the radius of curvature of the spherical surface. $ST = \frac{PS}{2}$, where SP is the mean diameter of the ring. This is measured by means of a pair of slide calipers used as inside calipers, the jaw correction being given with the instrument.

To determine R it only remains to measure the distance OT as accurately as possible. In the first place to make the ring concentric with a line through the axis of the screw an arm whose length is nearly equal to the radius of the ring is fitted on the top of the micrometer screw. When the ring has been adjusted, until it is concentric with the circle swept out by the arm as it is rotated, it is clamped with the set-screw below the ring.

To determine the pitch of the screw in terms of the standard scale on the slide calipers, a small piece of glass is mounted as shown to the right of figure 16. The small piece of plate glass is made to adhere to the larger piece by breathing on the surfaces before pressing them together. The point of the screw is brought into bare contact at M by turning the milled head on the axis of the micrometer screw. It requires a somewhat delicate sense of touch to determine when the point of the screw is barely in contact with the glass. A large number of readings of the position of the micrometer screw should be taken and averaged. The small piece of glass is then removed and the screw turned until its point is barely in contact with the upper plate, the readings of the micrometer screw being taken as before. The number of whole turns and fraction of a turn of the micrometer screw is divided into the thickness of the small piece of glass as measured with the standard scale of the slide calipers.

Determine the radii of curvature of the two faces of a convergent lens by means of the ring-spherometer. Report the number of the lens and the number of the spherometer as given by the label on the instrument.

Obtain on the wall an image of some distant object outside the room and measure with a meter rod the distance from the center of the lens to the image. This is a mean value of the focal length of the lens for white light. Call it f. From the equation $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$, where r_1 and r_2 are the radii you have just measured, calculate μ the mean value of the index of refraction of the glass for white light.

(*b*) **The Peg-Spherometer.** In this form of spherometer the micrometer screw is on a tripod, the legs of the tripod ending in fine steel points with which contact is made with the spherical surface. These three points should be equidistant from each other and a circle through them should have its center in line with the axis of the micrometer screw. Call the radius of this circle a and the average distance between the three points l. But the square of the side of an equilateral triangle is equal to three times the square of the radius of the circumscribing circle, therefore $l^2 = 3\,a^2$. Now a corresponds exactly to the radius of the ring in the case of the ring-spherometer. Therefore as we proved before $a^2 = \mathrm{OT}\ (2\mathrm{R} - \mathrm{OT}) = \frac{l^2}{3}$. OT is measured by placing the spherometer on the spherical surface and obtaining the reading of the micrometer screw when its point is barely in contact with the surface. The process is repeated using a piece of plane glass. The difference of the two readings gives the distance OT. The pitch of the screw is obtained as before by comparing the value of the thickness of a small piece of plate glass (in terms of revolutions of the screw) with its thickness as determined by the slide calipers. The student should examine this form of spherometer as well as the ring-spherometer. He should make at least one determination of curvature with it so that he may become familiar with the mechanical adjustments and with the calculation from the formula given above.

Tests of Plane Surfaces by Means of Interference Bands from Monochromatic Light. The planeness of surfaces may be tested very easily and with a high degree of precision if a glass plate with an accurately plane surface is available. The latter surface is placed on the surface to be tested and the two surfaces are viewed at an oblique angle by means of the light from a sodium burner which is placed on the opposite side of the surfaces from the observer. If the two surfaces are plane he will observe a series of parallel dark and bright bands. The distance between these bands will depend on the angle at which they are viewed and on the angle of inclination of the two surfaces to each other. If there is a spherical hump on the surface to be tested a series of Newton's rings will be seen in the neighborhood of the hump. If the change of curvature is sharp at this point the rings will be very close together. If there is only a slight change of curvature the rings will be far apart. If the hump is elliptical in section (considered parallel to the general surface) elliptical interference bands will be obtained and so on. The student may test the planeness of the surfaces of a piece of plate glass by comparing them with the surface of the black glass used in the Newton's rings experiment. The surfaces should be wiped with a piece of clean chamois leather to remove any grit from the surfaces.

Experiment 21. Measurement of Radius of Curvature of Spherical Surfaces by Optical Methods. (*a*) **Large Convex Mirror.** The large mirror is mounted on a stand placed near the straight edge of a table. The mirror is placed carefully against the edge of the table at the point B as indicated in figure 17. Two pins to which strings are attached are inserted in the wood at the edge of the table.

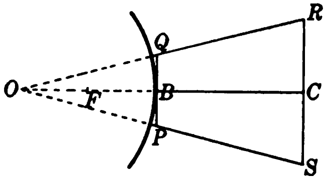


Fig. 17. Illustrates Measurement of Radius of Curvature of Large Convex Surface.

The strings and mirror are adjusted until the strings are in line with their own images. If the strings are still in the plane of the table this result means that the center of the spherical surface is in the same plane as the surface of the table. At a distance from the mirror indicated by BC in the figure a straightedge is laid on the table parallel to the line joining the two pins. Let the straightedge cut the strings in the points R and S. Then by similar triangles

$$\frac{RS}{PQ}=\frac{OC}{OB} \text{ and therefore } \frac{RS-PQ}{PQ}=\frac{OC-OB}{OB}=\frac{BC}{OB}.$$

Therefore by measuring RS, PQ, and BC, we can determine the radius of curvature OB. Rays parallel to CB would be reflected back as if they came from a point midway between O and B. This point (F in the figure) is therefore approximately the focus of the spherical mirror.

(*b*) **Small Convex Mirror.** Two incandescent lamp filaments at T and S respectively are taken as an object for the small convex mirror of center O. The rays of light that reach the spherical surface from T are reflected back as if they came from T_1. Similarly the image of S is S_1. The width of the image T_1S_1 cannot be measured directly, but it can be determined by putting a scale just below the apex of the mirror at M. T_1S_1 and the scale are both viewed with a telescope, the intersection of whose cross-hairs are in the line TS. Now the image of T, that is T_1, must lie on the line TO, for the ray that proceeds from T along this line will be reflected back along the same line. Similarly S_1 lies on the line OS. Therefore by similar triangles $\frac{T_1S_1}{TS} = \frac{OS_1}{OS}$. By a simple theorem in proportion

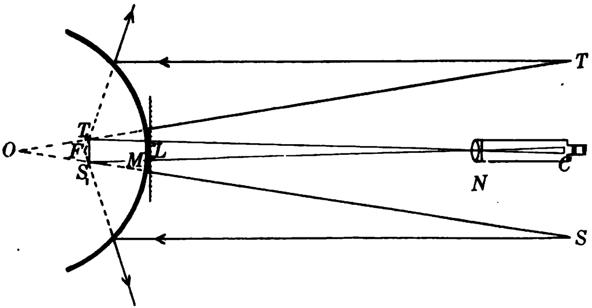


Fig. 18. Illustrates Measurement of Radius of Curvature of Small Convex Surface.

$$\frac{2\overline{T_1S_1}}{TS - 2\overline{T_1S_1}} = \frac{2\overline{OS_1}}{OS - 2\overline{OS_1}} = \frac{R}{SM},$$ where R is the radius of curvature of the spherical surface. Putting this equation in words, we have

$$\frac{\text{Radius of Curvature of Convex Mirror}}{\text{Distance from Objects to Mirror}} = \frac{\text{Twice Distance between Images}}{\text{Distance between Objects}-(\text{Twice Distance between Images})} \quad (1).$$

For approximate measurement of the radius of curvature the distance L read off on the scale at M (by means of the telescope with the micrometer screw attachment) is taken equal to T_1S_1 the distance between the images. Strictly $\frac{T_1S_1}{L} = \frac{NS_1}{NM}$, where N is the position of the objective.

The value of T_1S_1 thus obtained may now be put in equation (1) and an accurate value of R obtained.

In carrying out this method the student may mount a small lens with a convex surface (whose radius of curvature is to be tested) and at a convenient distance from it a telescope and scale. The scale is only used as a support for two incandescent lamps whose filaments are the objects T and S.

The telescope is focussed on the images of the incandescent filaments in the mirror.

A scale is set up just below M and an image of this scale is obtained in the plane of the cross-hairs alongside the image of the incandescent filaments. By means of the micrometer screw and attached cross-hair, T_1S_1 is obtained in scale divisions (of the scale at M) though subject to a slight correction on account of the slightly different distances of T_1S_1 and the scale from the telescope. The uncorrected reading has been called L. Using this for T_1S_1 obtain an approximate value for the radius of curvature of the convex surface of the lens.

This method is used in ophthalmology for determining the radius of curvature of the eyeball.

(*c*) **Concave Mirror.** The focal length of a concave mirror is easily obtained by getting with its aid a real image of a distant object (trees outside the room for example). The mirror may be held at about its focal length from the window casing and an image of the distant trees obtained on the white plaster. If this is not practicable a piece of white cardboard may be used. The distance from the mirror to the image is equal to the focal length. The radius of curvature is equal to twice this distance. Again the mirror may be mounted on a standard of an optical bench. On another standard of the same bench a large piece of white cardboard is mounted and a straight slit cut in it at about the height of the mirror. This slit is illuminated by means of a lamp on the opposite side. The mirror is then adjusted until an image of the slit is obtained on the screen near the slit. The distance from the slit to the mirror is then equal to the radius of curvature.

Experiment 22. Measurement of the Focal Length of a Thin Lens by the Method of Conjugate Foci. Spherical Aberration. Chromatic Aberration. We have already shown that for a thin lens

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v},$$

where f is the focal length of the lens, u the object distance and v the image distance. In order to measure the object and image distances with as great accuracy as possible, the lens, the object screen, and the image screen are mounted on an optical bench. This has a heavy base of iron and is provided with iron set-screws (for holding the standards firmly) and with brass clamps for the lens and the screens. To obtain a suitable object screen a piece of thin glass plate is ground slightly on one surface. This surface is ruled with one series of equidistant parallel lines with a black pencil. Another series of black lines is ruled at right angles to the former. The ground surface of the glass is turned toward the lens. The smooth surface is

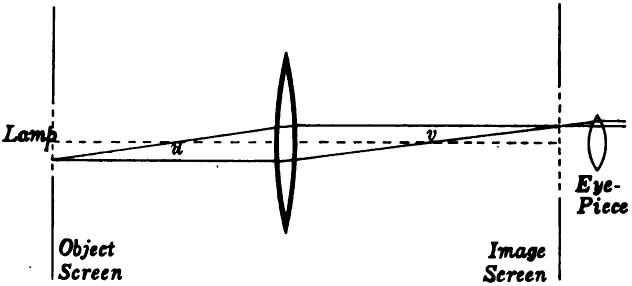


Fig. 19. Illustrates the Method of Conjugate Foci.

turned toward the source of illumination which may be an incandescent lamp or luminous gas flame. The ground glass surface serves as a luminous object while shutting from view the bright incandescent filament. The image screen may be conveniently made of fine wires stretched across an opening in a screen. The eye-piece shown in the figure is kept focussed on the image screen. This is intended to be used in a parallax test to see whether the wires are in the same plane with the image of the ground glass. To make the adjustment approximately it is only necessary to stretch a piece of white paper across the wires and move the lens and image screen until a clear image of the object is obtained on the white paper. The lens is now fixed, the white paper removed, and the position of the wire screen adjusted until no parallax is detected when the image and the wires are viewed through the eye-piece. In making the parallax test the eye is moved to and fro in front of the eye-piece. It is possible that motion between the image and the wires will occur for every position of the image screen. This would be due to spherical aberration of the lens. The edge of the lens for example may have a different focal length from the central portion of the lens. This may be very easily tested by using a paper diaphragm to shut off the light from any desired portion of the lens.

Make a diaphragm of a piece of white paper with an opening a few millimeters in diameter and mount it over the lens clamp so that the light from the object passes through only the central portion of the lens. It will now be possible to make very accurate settings of the image screen. Obtain several widely different pairs of values of u and v, varying the ratio $\frac{u}{v}$ from $1/5$ to 5 if the length of the optical bench permits. This ratio gives the relative size of object and image. The data obtained may be combined as follows. Rule off two lines (OY and OX) of as great length as your note-book will allow, placing the point of intersection O near the lower left-hand corner of the page. Mark off along OX on a convenient scale the values of u, and similarly on OY the values of v. Connect the points which correspond by straight lines. It will probably be found that these lines pass approximately through a point. This point is equidistant from OX and OY, the perpendicular distance from each being f. In determining the value of f however, the results obtained from the observations where u and v are nearly equal are likely to be the best. From the point of intersection given by these values determine the focal length of the central portion of the lens. Repeat the experiment with the diaphragm near the edge. If the two focal lengths come out differently it is an indication of *spherical aberration* in the lens.

To test the lens for *chromatic aberration* obtain an image on the image screen such that v is equal to or somewhat greater than u. Remove the ground glass surface and put in its place a screen made partly of tin foil which has been pierced in several places with a fine needle. If a diaphragm is used in front of the lens it should have a large opening to provide the necessary intensity of light at the image screen. When the student now attempts to focus by the parallax method he will probably be able to focus for only one color for any given opening in the tin foil. He may get an image of this opening in red light for example, while the image will be surrounded by fringes of yellow, green, etc., which are out of focus. These show very large movements when the eye is moved from side to side in making the parallax test. The outer edge of the fan-shaped fringe is violet, the light of that color being most out of focus. By pushing the screen a little towards the lens it is possible to obtain an image in green light that stands the parallax test, while the red and violet are seen out of focus on opposite sides of the green opening.

Obtain the focal lengths of the central portion of the lens for two of the following colors of the spectrum: red, green, blue, violet.

Experiment 23. Measurement of the Focal Length of a Divergent Lens. Combination of Lenses. Rays of light whose directions are parallel to the axis of a divergent lens are bent by it so that their directions after leaving the lens come from a point approximately. The distance from this point to the lens is the focal length of the latter. If its value is put in the equation

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f},$$

f must be represented by a negative number. If the incident rays come from a point in a given object the value of u is always positive. In this case it is evident from the equation that the value of v will be negative. We see that this must be the case if we reflect that the rays are already divergent when they strike the lens and that it diverges them more. This means that the image is virtual. A divergent lens therefore always gives a virtual image of a real object. Only a real image can be obtained on a screen, but it is possible, as we shall see, to determine the position of the virtual image by a parallax test. The focal length of the lens may be measured directly by means of a parallax test as follows.

(*a*) **Direct Measurement.** The lens is mounted in a long trough whose sides have been blackened. Some distant object is observed through the lens, preferably one that has straight vertical lines such as a chimney or a telegraph pole. A small image of the distant object may be obtained within the limits of the trough if the focal length of the lens is not too great. A scarf pin is stuck in the trough so that it is perpendicular to the axis of the lens. The eye is now moved from side to side to see whether there is any relative motion between the image of the distant object and the scarf pin as seen over the lens. If the pin is farther away than the image, then it will appear to move to the right relatively to the image when the head is moved to the right, and *vice versa.* When no relative motion between the image and the pin is obtained the distance from the pin to the lens is equal to the focal length of the lens.

(*b*) **Close Combination with a Convergent Lens.** If the focal length of each of the lenses is large as compared with the thickness of the combination, the focal length of the latter may be easily calculated. Let the focal lengths of the divergent lens and the convergent lens be f_1 and f_2 respectively. Let light parallel to the common axis of the two lenses fall first on the divergent lens. The rays will be bent by it as if they came from a point f_1 centimeters from the combination. This forms the object for the convergent lens. Therefore we have $\frac{1}{f_1} + \frac{1}{v} = \frac{1}{f_2}$. But v the image distance is in this case very nearly equal to the focal length of the combination. Calling it F, we may write $\frac{1}{F} = \frac{1}{f_2} - \frac{1}{f_1}$. The combination of the two lenses will be a convergent or divergent lens according as f_2 is less or greater than f_1.

In making the determination of the focal length of the divergent lens in this manner, it is better to choose a convergent lens whose focal length is considerably less than that of the divergent. Then a real image of a distant object may be obtained from the combination, and the distance from the image to the combination will give F in the above formula. Next determine the focal length of f_2 in the same manner. These two measurements give the data for calculating f_1.

(*c*) **Combination with a Convergent Lens which is at the Distance *d* from the Divergent Lens.** Suppose that as before the light is incident first on the divergent lens. The light is bent as if it came from a point f_1 centimeters from the divergent lens and therefore $f_1 + d$ centimeters from the convergent lens. Therefore $\frac{1}{f_1 + d} + \frac{1}{v} = \frac{1}{f_2}$. Here v is the distance of the last lens from the focus of the combination. Call this B_{12}. Then $\frac{1}{B_{12}} = \frac{1}{f_2} - \frac{1}{d + f_1}$. Therefore a real or a virtual image will be formed according as f_2 is numerically less or greater than $d + f_1$.

Select a convergent lens that combined with the divergent lens a few centimeters away from it gives a positive combination when parallel light is incident on the divergent lens. This may be tested by getting an image of some distant object on the wall. Measure the distance B_{12}. Measure also f_2 and d. Calculate f_1 the focal length of the divergent lens.

If the length B_{12} is between 20 cm. and 100 cm., it may be conveniently measured on the optical bench with the aid of a plane mirror. O (fig. 20) represents a slit in a screen of white cardboard. The slit is illuminated by means of a luminous gas flame on the side of the slit opposite from the lens. The position of the combination of lenses is adjusted

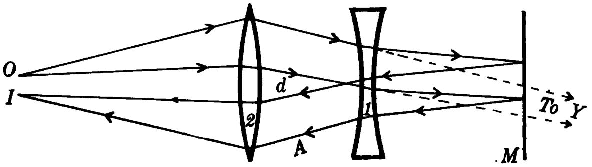


Fig. 20.

until a clear image I is obtained that moves when the mirror M is moved. The light between the divergent lens and the mirror is then approximately parallel. The parallelism of the light may be tested by moving the mirror farther away from the divergent lens and nearer to it alternately. If the light is parallel there will be no change in the definition of the image.

The distance from the convergent lens to OI now gives the value of the distance B_{12}. From this, from the previously determined value of f_2, and from your measurement of the length *d*, calculate the value of f_1.

(*d*) The focal length of the divergent lens may again be measured directly with the arrangement shown in figure 20. After the adjustment which makes the light parallel between the divergent lens and the mirror has been obtained and the distance *d* and the thickness of the divergent lens noted, this lens may be removed. A real image will now be obtained from the convergent lens. Let this image be situated at Y. Replace the divergent lens after the position of Y has been carefully noted. The distance from Y to the divergent lens is equal to the focal length of the latter.

Determine the focal length (of the divergent lens furnished) by as many of the above methods as possible.

Experiment 24. Study of the Terrestrial Telescope. The terrestrial telescope does not invert the positions of objects as does the astronomical telescope. There must be therefore two real images formed inside the terrestrial telescope. The approximate position of these is shown in figure 21.

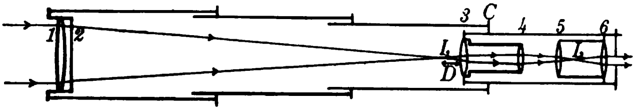


Fig. 21. Formation of Images in the Terrestrial Telescope.

While dissecting the telescope the student should have at hand a box in which he can put the lenses in order as they are taken out. Before replacing the lenses the dust should be carefully wiped off with a clean cloth after the glass surface in each case has been slightly moistened by breathing on it.

First focus the telescope on some distant object. This is best done through the open window.

Remove the objective and separate the two lenses. What difference in the color of the lenses do you observe? Which is the crown glass lens? Which the flint glass lens? Measure the focal length of the convergent lens directly by obtaining with it a real image of a distant object. Measure the focal length of the divergent lens by obtaining a virtual image of a distant object and locating it by test for parallax. Calculate the focal length of the combination, using the equation $\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2}$. Measure the focal length of the combination by obtaining a real image of a distant object. Compare the focal length thus obtained with the calculated value.

It is now desirable to find out how far the real image (formed by the objective) lies from the eye-piece. The distance between lenses 2 and 3 (when the telescope is focussed on a distant object) should now be measured by unscrewing the extensible tube from the short tube that clasps the eye-piece. This is at the position indicated by C in figure 21. The short tube with the eye-piece in it is placed on the table beside the remainder of the instrument in such a way that the distance between lenses 2 and 3 can be measured. By subtracting from this the focal length of the combination the distance of the real image from 3 may be obtained. This result may be checked by focussing the eye-piece (clamping it first firmly) on a plane object and measuring the distance from 3 to the object. This may be called the distance of distinct vision. Call this D. The distance D plus the focal length F of the objective should be equal to the distance between objective and eye-piece (between 2 and 3) when the telescope is focussed on a distant object, that is $D + F = 2 - 3$.

Measure the focal lengths of the lenses of the eye-piece (3, 4, 5, 6) and the distances between them. Starting with an object at the point I_1 calculate where the image due to the combination 3, 4, 5 would come. It will probably be found that it comes nearly at the focal point of the lens 6. If this is the case the light will come from the lens 6 nearly parallel.

Make a drawing of the telescope on a scale of one-half indicating the positions of the images I_1 and I_2, the latter being found from your calculation.

Measure the magnifying power of the telescope for an object at a distance of twenty to thirty feet. This is conveniently done by using a board containing a number of black and white squares placed alternately along the length of the board. The observer then notes the number of squares (as seen by the naked eye) that are covered by one square as seen through the telescope.

Hold the objective of the telescope toward the sky or a bright light in the laboratory. If a meter rod be now held outside lens 6 an image of the objective opening (given by the eye-piece) may be obtained. This is called the ocular circle. The diameter of the objective divided by the diameter of the ocular circle gives a rough indication of the magnifying power of the telescope for a distant object.

Report the number of the telescope on which your measurements were taken.

Experiment 25. Measurement of the Angles of a Prism by Means of a Spectrometer. Measurement of

Refractive Index of Material of Prism for Light of Different Frequencies. The principal parts of a spectrometer are the angular scale, the collimator tube, the receiving telescope, and the spectrometer table. These are all mounted on a very heavy base provided with three leveling screws. The collimator tube contains the slit which is an extremely important part of the apparatus. The edges of the slit should be accurately parallel to each other. To prevent corrosion of the edges from salt vapor, a piece of thin mica should be mounted in front of the slit. At the other end of the collimator tube there is an achromatic lens which is used to make the light that comes from the slit parallel before it it strikes the prism on the spectrometer table. The light is refracted by the prism and enters the observing telescope. If the latter has been adjusted for parallel light, an image of the slit will be obtained in the plane of the cross-hairs. For the purpose of the measurements suggested above the following adjustments are necessary.

The axes of the telescope and collimator should be perpendicular to the axis about which the telescope rotates. This axis should coincide with the axis of the circular scale. The latter adjustment belongs primarily to the maker of the

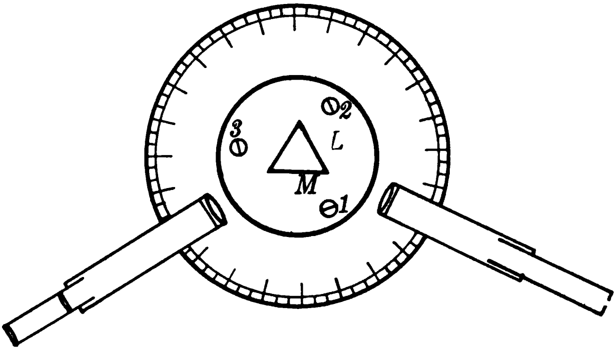


Fig. 22. Principal Parts of the Spectrometer.

instrument. Errors due to the fact that the center of the scale lies off the axis of rotation of the telescope may be partially corrected by taking readings on the scale 180° apart for each position of the telescope. We shall assume in this course that the axis, about which the telescope rotates, coincides with the axis about which the spectrometer table rotates. In accurate work the axis of the telescope is made perpendicular to its axis of rotation as follows. A plane-parallel plate whose faces have been lightly silvered is mounted with its plane as nearly as possible in the axis of rotation of the table. An image of the cross-hairs from one of the parallel faces is obtained (by adjustment of the telescope) in the plane of the cross-hairs, and the intersection of the cross-hairs is made to coincide with its own image. The telescope is turned through 180°. If the image does not coincide in this position with the object, the deviation is removed half by inclination of the plate and half by inclination of the telescope. By repeating this process the axis of the telescope may be made perpendicular to its axis of rotation. As this adjustment takes considerable time it will be performed by the assistants in the course before the student comes into the laboratory. Great care should be taken not to disturb this adjustment. The student is asked however to perform the following adjustments himself.

To focus the telescope for parallel rays it is rotated (about the axis through the center of the scale) into a position where it can be sighted on some distant object outside the window. The eye-piece is focussed on the cross-hairs, and then the tube containing both eye-piece and cross-hairs is pulled out or pushed in until a clear image of the outside object is obtained. The cross-hairs are turned until they make an angle of 45° with the vertical.

Turn the instrument so that the collimator slit is toward the window, or illuminate it with artificial light. Rotate the telescope and adjust the position of the collimator in a vertical plane until the axis of the collimator is in line with the axis of the telescope. This adjustment is completed when the image of the middle of the collimator slit coincides with the intersection of the cross-hairs in the observing telescope. The latter result also requires the moving of the slit in or out, until its image is sharp and in the plane of the cross-hairs as indicated by the parallax test.

Place a triangular prism on the table in such a position that the image of the slit obtained by reflection from either of the faces L, M (see fig. 22) may be brought to a focus in the plane of the cross-hairs of the observing telescope. To observe these two images in succession will require the rotation of the telescope through an angle equal to twice the angle between the faces L and M. It is convenient also to have one of the faces of the prism, the face M for example, at right angles to a line joining two of the levelling screws, 1 and 2 for example.

To level the prism, that is to adjust it so that its faces are perpendicular to the plane swept out by the telescope in a rotation around its own axis, the prism is moved by means of levelling screws, 1, 2, 3 until the reflection of the middle of the slit from each of the faces L, M is brought to a focus at the intersection of the cross-hairs when the observing telescope is rotated into the proper position. Let the telescope be placed in position to get the reflection of the light from the slit as reflected from the face M. The middle of the slit is marked by means of a fine wire stretched across it at right angles. By turning screws 1 or 2 adjust the face M until the image of the middle of the slit coincides with the intersection of the cross-hairs. Turn the telescope until the reflection from the face L is obtained, and by adjusting with the screw 3 get the image of the middle of the slit again on the intersection of the cross-hairs. Since the face M has been placed approximately at right angles to a line joining 1 and 2 the motion caused by turning the screw 3 should not disturb much the adjustment previously made for the face M. By repeating the process the prism may be very accurately levelled.

Make the slit very narrow. Rotate the telescope until the image of the slit as obtained by reflection from the face L lies very accurately on the intersection of the cross-hairs, the final adjustment being made by means of the slow motion screw (tangent screw). Take readings on the scale for the position of the telescope by means of the two verniers attached to the telescope. Rotate the telescope and obtain a similar set of readings for the case where the reflection is from the face M. The difference between the mean readings on the scale for the two faces gives twice the angle between the faces L and M.

Similarly measure the other angles of the prism. Check your results by comparing the sum of the three angles with 180°.

Illuminate the slit with light from a sodium burner. Obtain an image of the slit by refracted light through one of the angles of the prism. For preliminary adjustment this may be done with the eye without the aid of the telescope. Turn the prism slowly in a direction that carries the observer's eye toward a line through the axis of the collimator; that is the prism is turned in such a direction that the deviation of the rays of light from the slit becomes less. It is found that the deviation decreases to a minimum value, the deviation increasing again as the prism is rotated farther in the same direction. Having obtained the position of minimum deviation without the telescope, complete the adjustment with the aid of the telescope, leaving the image of the slit on the intersection of the cross-hairs. Read the position of the telescope on the scale. Remove the prism and rotate the telescope until the direct image of the slit is obtained at the intersection of the cross-hairs. Read again the position of the telescope. The difference between the two readings gives the angle of minimum deviation. Call it δ. Call the refracting angle θ. Then as has been proved on page 65, the index of refraction of the glass of the prism for sodium light in air is given by the equation

$$n = \frac{\sin \frac{1}{2}(\theta + \delta)}{\sin \frac{1}{2}\theta}.$$

Remove the sodium burner. Put some lithium chloride on a loop at the end of a steel wire and measure the angle of minimum deviation for lithium red. With another steel wire that has some calcium chloride on the loop at the end obtain the angle of minimum deviation for the green band. If time permits the student may do the same with the red line given by potassium and the blue line given by strontium. The following table gives the wave-lengths of these lines or bands in air.

SALT IN GAS FLAME.	DESCRIPTION.	WAVE-LENGTH IN AIR.	FREQUENCY.
			Trillion a second
Potassium Chloride or Nitrate......	Deep red, double...	768×10^{-6} mm.	391
Lithium Chloride..	Single line, red.....	671	447
Sodium Chloride...	Double line, yellow.	589	509
Calcium Chloride..	Green band........	553	543
Strontium Chloride	Single line, blue....	461	651

It is not probable that the student will be able to resolve the double potassium and sodium lines. With the apparatus at his disposal they will appear as single lines. Report the refractive indices of the glass of the prism for as many as possible of the frequencies given in the table.

Experiment 26. Calibration of a Spectroscope by means of Arc Spectra. The spectroscope differs from the spectrometer in that there is no angular scale that extends completely around the prism table. The telescope moreover is only movable to a limited extent. This motion is brought about with a tangent screw. The field is only wide enough (in the instruments in this laboratory) to enable the observer to get the luminous spectrum from the sun within the field of view. A small scale containing not more than 200 divisions is placed within a third arm of the instrument. An image of this scale is brought to the plane of the cross-hairs of the telescope alongside the spectrum lines which are to be studied. The sharp spectrum lines obtained in the field of the cross-hairs are images of the slit in monochromatic light. The prism has been placed by the maker in the position of minimum deviation for sodium light, the position of the axis of the collimator being supposed to remain fixed.

Unscrew the telescope and focus it for parallel light, having previously focussed the eye-piece on the cross-hairs. Replace the telescope and adjust the slit until a vertical image of the latter (in sodium light) is obtained in the plane of the cross-hairs of the telescope. Adjust the position of the scale in the third tube until a clear image of it is obtained in the plane of the cross-hairs of the telescope, the scale lines being parallel to the image of the slit. Move the telescope with the tangent screw until the yellow sodium line is coincident with the division 50 on the image of the scale. In using the arc to obtain spectra it is desirable to screen the spectroscope well. In making the adjustments with the arc the student will find it advisable to use dark glasses in order to avoid injuring the eyes. A separate pair of carbons is kept for each salt experimented with. When not in use these carbons are kept soaking in solutions of the respective salts with which they are intended to be used. Great care should be taken to avoid contaminating any one of these salt solutions with any other. When the carbons are very close together the student will probably observe only the continuous spectrum from the solid incandescent carbon. If one of the carbons is pulled out a little from the other (the student should be careful not to touch both terminals at once) a line spectrum is obtained from the flaming arc.

Take readings on the scale of as many as possible of the lines given in the table below, the sodium line being kept on division 50 of the scale. The wave-lengths and frequencies are given in the same units as in the case of the flame spectra.

SALT.	DESCRIPTION OF LINE.	WAVE-LENGTH.	FREQUENCY.
			Trillion a second
Lithium Chloride	Medium red, very intense.	670.8	447
" "	Orange, very intense......	610.4	492
" "	Green, faint only visible after fresh salt is applied	497.2	603
" "	Blue, very intense........	460.2	652
" "	Indigo, moderately intense, not sharply defined.....	413.2	726
Thallium Chloride	Very intense green line...	535.1	561
Strontium Nitrate	Indigo, very intense......	421.6	711
" "	Violet, very intense.......	407.8	735
Calcium Chloride	Violet, very intense.......	396.9	756
" "	" " "	393.4	763

When the positions of a number of these lines has been read on the spectroscopic scale, the wave-lengths of the lines in air are plotted as ordinates and the corresponding spectroscope readings are plotted as abscissæ. The student is advised to plot first the sodium yellow line and then the lithium red, the lithium orange, and the lithium blue. These lines are very intense and almost unmistakable. If the curve be drawn through these points first it will aid in determining some of the other lines which the student is more likely to mistake. The calcium lines of wave-lengths 396.9 and 393.4 are the same as the H and K lines of the solar spectrum. They frequently lie outside of the field of the spectroscopes used in this laboratory. The student should ask for assistance if he is unable to recognize the above lines in the indigo and violet.

After the curve has been plotted the student may determine approximately the wave-lengths of the more intense lines in the red, orange, yellow, green and blue portions of the strontium arc spectrum.

Experiment 27. Determination of the Wave-Lengths of Fraunhofer Lines by means of the Spectrometer and Grating. The gratings used in the laboratory are replicas of a Rowland grating. They contain many thousand lines to the centimeter. From the many thousand openings in the surface of the plate bright and dark bands may be obtained on a screen. They are similar to those obtained in Young's experiment except that the angular dispersion of the bands is much greater. The diffraction spectra of the grating may be readily seen by holding the grating at right angles to the sun's rays. The greater part of the sunlight will go on in its original direction after passing through the grating. The diffracted light gives two spectra (that can be easily seen on a dark background) on each side of the central patch of light that has come directly through. We may call these spectra the spectrum of the first order on the right, the spectrum of the second order on the right, the spectrum of the first order on the left, etc. The conditions for the formation of a spectrum of the first order may be seen from figure 23. The double-pointed arrows show the direction of the light that is incident on the grating and also that of the light which is not diffracted. The light waves that are diffracted at the openings of this grating in general interfere, but if we consider a wave-front ABK such that the average distance of each successive opening from the wave-front differs by one wave-length from the distances of the nearest openings to the same wave-front, then the light from each opening will reinforce the light that comes from the other openings. In the present experiment the light that comes in a direction perpendicular to ABK is collected by the objective of the observing telescope of the spectrometer. The latter is focussed for parallel light. In order to obtain in the telescope the rays that will form the first diffracted image for light of wave-length λ, the telescope must be rotated until it is perpendicular to ABK. If

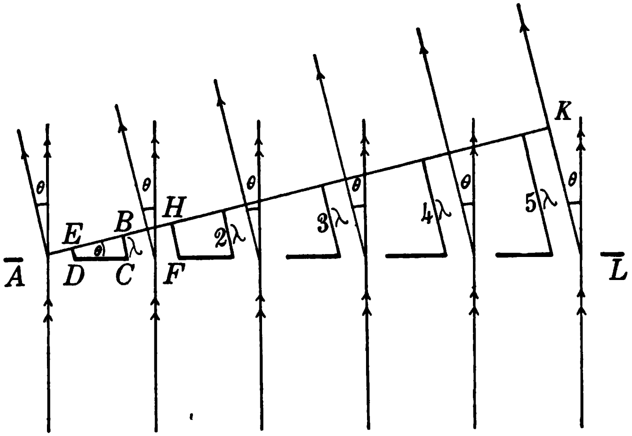


Fig. 23. Diffraction from a Plane Grating.

it is originally directly opposite the collimator tube, the telescope must be turned (to get the first diffracted image) through an angle equal to that between ABK and ACL, that is the angle θ. But $\sin\theta = \frac{BC}{AC} = \frac{\lambda}{a}$, where a is the mean distance between two consecutive openings. The angle θ is easily measured with the spectrometer. If we know the value of λ (as in the case of sodium light) we may calculate the mean distance between the openings and consequently the number of openings to a centimeter.

The adjustments of the spectrometer in the present experiment are much like the preliminary adjustments in experiment 25. The observing telescope is focussed for parallel light, the slit of the collimator is focussed on the cross-hairs of the observing telescope. The grating is placed approximately perpendicular to the axis of the collimator. Sunlight is reflected into the room by means of a mirror outside the window. A large convergent lens is used to focus the light from the mirror on the slit of the collimator. The slit of the collimator should be parallel to the lines of the grating. Place a darkened photographic plate over the slit to diminish for a moment the intensity of the light and rotate the observing telescope until the intersection of its cross-hairs is on the image of the slit. Read the position of the telescope on the scale. Rotate the telescope to the left until the intersection of the cross-hairs is on one of the D lines (two sodium lines in the yellow) and read the position of the telescope again. The difference between the two readings is the angle of diffraction for the given wave-length on the right. Determine the angle of diffraction for the same line on the left. If the two angles of diffraction (to the right and to the left respectively) are nearly equal, the mean of the two may be taken. If they differ much the grating will have to be turned until the two angles of diffraction are nearly equal. When this is the case the mean angle of diffraction (for the first diffracted images to the left and the right respectively) is taken. The sine of the angle that has been determined in this way divided by the wave-length of the light used (expressed in centimeters) given the number of lines per centimeter. This is called the grating constant.

Measure the wave-lengths of the C line, of the line of the *b* group that is nearest the red, and of the F line. These lines may be easily recognized by comparing the spectrum as observed in the plane of the cross-hairs with a map of the solar spectrum. Report the number of the grating used and the value of the constant obtained for it.

Photometry. The ether-waves from a luminous particle spread themselves out in an isotropic medium equally in all directions. If there is no absorption the same total amount of light from a given luminous point passes through each spherical surface that is concentric with the luminous point. It follows that the light that falls on a square centimeter of one of these spherical surfaces is inversely proportional to the square of the distance from the luminous point. If two sources give equal intensities on a small surface (that lies between them so that the light from each source strikes at the same angle), then we have

$$\frac{I_1}{d_1^2} = \frac{I_2}{d_2^2},$$

where I represents intensity of source and d distance from the source to the screen. This equation is used in the experiment with the Bunsen photometer which follows.

The old standard source of light, the candle-power, was a spermaceti candle ⅞ inch in diameter and burning 120 grains an hour. A more exact unit is the Hefner candle, which is the 40 millimeter high flame from a primary isoamylacetate lamp burned with a wick 8 millimeters in diameter out of a German silver tube whose wall is 0.15 millimeter thick.

Experiment 28. Measurement of Candle-Power with a Bunsen Photometer. For a description of the Bunsen Photometer see Watson, pp. 509-510 and Hastings and Beach, *General Physics,* p. 719. As a comparison source we use in this experiment the illumination given by an Argand burner through an opening in a screen. This opening is a few millimeters in diameter and is placed opposite the central portion of the luminous flame. Compare this source with a Hefner lamp which is made up and adjusted as described above. Place the two sources about two meters apart in line with the guide on which the carriage containing the Bunsen screen runs. Move the screen until both sides are equally illuminated. Calculate the candle-power of the comparison source. Put out the amylacetate lamp and use the comparison source which has now been standardized during the rest of the experiment.

Measure the candle-power of a fish-tail burner under a moderate pressure of gas turning it at angles 90°, 60°, 45°, and 30° to the line joining the centers of the two sources. Plot the observed intensities as ordinates and the corresponding angles as abscissæ.

Measure the candle-power of an incandescent lamp.

Comparisons of this source are only satisfactory when conducted between sources which have about the same proportions of the different colors of the spectrum.

Experiment 29. Measurement of Coefficient of Absorption of Different Materials for Light of a Particular Wave-Length. Measurement of Reflecting Power. Figure 24 shows a photometer of the Cornu type which is very convenient for the measurements of this exercise. This photometer does not compare the total intensities of light given off by different sources, but compares the intensity given off per unit area of one source with the corresponding quantity for another source. L is a gasoline lamp which is used as a comparison source throughout the experiment. An image of a flame from this lamp is obtained in front of the eye-piece by means of the lens L_1 and the mirror M. Beside it is obtained an image of an outside source by means of the light which comes through the objective. In front of the objective is a cat's-eye opening whose diagonal length may be varied, and read by means of the scale S. By varying the size of this opening the images of the comparison lamp and of the outside source may be made of the same intensity when viewed through a piece of red glass that lets through approximately monochromatic light. This glass is placed between the eye and the eye-piece. Suppose that *s* is the length of the diagonal opening when the two images are of the same intensity, there being no absorbing glass in

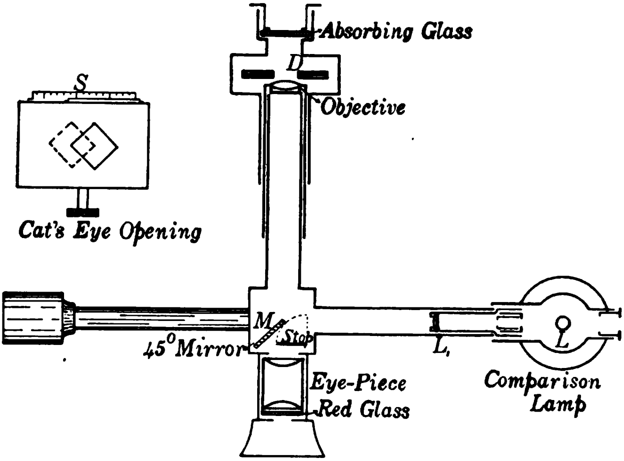


Fig. 24. Photometer for Comparison of Intrinsic Intensities of Sources of Light.

front of the objective. Let s_a be the corresponding reading when there is an absorbing glass, the source outside the objective being kept constant. Then the intensities of the light that comes through unit area of the objective opening in these two cases are proportional to $\frac{1}{s_2}$ and $\frac{1}{s_a^2}$ respectively. The coefficient of transmission is

$$\frac{\frac{1}{s_a^2}}{\frac{1}{s^2}} = \frac{s^2}{s_a^2},$$

if the absorbing glass be of unit thickness.

Determine the absorption of a piece of glass that has been covered with a light layer of shellac. Of course the result obtained only applies to the light (of a particular range of frequencies) which can pass through the red glass near the eye-piece.

The reflecting power of a plane surface for a given angle may be determined by comparing the intensity of the light obtained directly from a given source and that of the light reflected from the surface at the given angle, the total path of the light being made the same in the two cases. This may be done by moving the photometer. Taking the light that comes directly as of unit intensity, plot the percentage of light reflected against the angle of incidence on the reflecting surface.

A fuller description of the photometer shown in figure 24 will be found in an article on optical pyrometry by Waidner and Burgess, *Bulletin of the Bureau of Standards,* Washington, Vol. I, No. 2.

Application of Photometry to the Measurement of High Temperatures. The photometer which has just been described is used in the measurement of the temperatures of furnaces and finds considerable application in metallurgy. The temperature of the Bessemer furnace at different parts, for example, may be measured quickly by the workman by noting the reading of the cat's-eye opening which gives equality of the intensity of the image of the hot body and that of the comparison lamp. Tables are prepared which give the relation between the readings of the cat's-eye opening and the temperature of the hot body. Measurements of high temperatures with optical pyrometers are performed in the course on heat measurements given at this university.

The Photographic Objective. Achromatic Combinations of Lenses. A good photographic objective has to comply with a very large number of requirements, varying with the kind of photography in which it is to be used. Speaking generally there must be a point to point correspondence between the object and the image. Objectives that are not free from astigmatism produce line images of luminous points that lie considerably off the axis of the objective. Again since photographic plates are nearly always prepared on flat surfaces of glass the image surface given by the objective should be flat. Objectives cannot be made from *lenses* so that they will be absolutely achromatic. Concave mirrors give achromatic images. Lens objectives may be made however so that they will have the same focal length for two colors of the spectrum, such as red and blue for example. For more definite statement of the requirements, the C and F lines of the solar spectrum are sometimes chosen. The requirement that the objective shall be free from the various kinds of spherical and chromatic aberration cannot be met by a single lens of a given material. Two or more lenses at various distances apart are required. This gives a large number of variables in the radii of curvature of the lens surfaces, the refractive indices of the materials, and the distance between the lenses. The greater the number of these variables the more possible it becomes to satisfy all the requirements.

An objective may be made of two lenses close together that will satisfy the most important requirements if the aperture angle be not too large. One of these lenses must be convergent and the other divergent. They must also be made of two different kinds of glass. The focal length of a lens for light for which the glass has the refractive index μ (the radii of curvature of the two surfaces being r_1 and r_2) is given by the equation

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$= \left(\frac{\mu - 1}{\mu_D - 1}\right)(\mu_D - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{\mu - 1}{\mu_D - 1} \cdot \frac{1}{f_D},$$

where μ_D is the refractive index of the glass for sodium light and f_D is the focal length of the lens (made of this glass) for sodium light. Suppose this lens to be convergent. If μ' and f' be understood as referring to a divergent lens of the second kind of glass, the focal length of the combination for the light of the C line is given by

$$\frac{1}{F_C} = \frac{\mu_C - 1}{\mu_D - 1} \cdot \frac{1}{f_D} - \frac{\mu'_C - 1}{\mu'_D - 1} \cdot \frac{1}{f'_D}.$$

For blue light of the F line the focal length of the combination will be given by

$$\frac{1}{F_F} = \frac{\mu_F - 1}{\mu_D - 1} \cdot \frac{1}{f_D} - \frac{\mu'_F - 1}{\mu'_D - 1} \cdot \frac{1}{f'_D}$$

When the combination is achromatic we have $F_F = F_C$. Putting the difference between the right-hand members of the two equations equal to zero we have

$$\frac{\mu_F - \mu_C}{\mu_D - 1} \cdot \frac{1}{f_D} = \frac{\mu'_F - \mu'_C}{\mu'_D - 1} \cdot \frac{1}{f'_D}. \quad (1)$$

Therefore the focal lengths of the two lenses must be proportional to the values of the quantity $\frac{\mu_F - \mu_C}{\mu_D - 1}$ for the respective kinds of glass. The numerator of this expression is called the mean dispersion, the denominator is the refractivity for sodium light. The following table gives values of these quantities for a number of materials. The different kinds of glass referred to in the table are all Jena glasses.

SUBSTANCE.		μ_D	$\mu_F - \mu_C$	$\frac{\mu_D - 1}{\mu_F - \mu_C}$
Fluor-Spar		1.4338	0.00446	97.3
Canada Balsam		1.526	0.0227	41.5
Diamond		2.4173	0.0251	56.6
Aniline		1.5863	0.0248	23.6
Water		1.3337	0.006	55.6
Silver Iodide		2.1816	0.123	9.6
JENA GLASSES.				
NUMBER.	DESCRIPTION OF GLASS.			
O 225	Light Phosphate Crown	1.5159	0.00737	70.
S 30	Dense Barium Phosphate Crown	1.5760	0.00884	65.2
O 40	Silicate Crown	1.5166	0.00849	60.9
O 1209	Densest Baryta Crown	1.6112	0.01068	57.2
O 726	Extra Light Flint	1.5398	0.01142	47.3
O 376	Ordinary Light Flint	1.5660	0.01319	42.9
S 57	Densest Silicate Flint	1.9626	0.04882	19.7

If any two of the glasses in the above table be used to form an achromatic combination their powers (reciprocals of focal lengths) must be respectively proportional to the numbers given in the last column. Hence the convergent lens of the pair must be made from the glass which has the higher value of

$$\frac{\mu_D - 1}{\mu_F - \mu_C}.$$

Since the condition for achromatic combination only gives us one equation among the four radii of curvature together with the condition that a real image must be formed, we still are able to vary the radii of curvature with the object of eliminating spherical aberration as much as possible and of producing a flat image surface. The latter condition requires that $\mu_1 f_1 = -\mu_2 f_2$, where the subscript 1 refers to the lens by which the light enters and the subscript 2 refers to the lens by which the light leaves the objective. This equation may be written

$$\frac{\mu_1}{\frac{1}{f_1}} = \frac{\mu_2}{\frac{1}{-f_2}}. \quad (2)$$

In order then to get a flat field the refractive indices of the two glasses must be proportional to their focal powers. To get an achromatic lens with a flat field it is necessary that the glass which has the higher value of $\frac{\mu_D - 1}{\mu_F - \mu_C}$ should also have the higher value of μ_D.

As an example let us calculate the focal powers of the lenses which will make an achromatic combination of 9 dioptres when the component lenses are made of the silicate crown glass "O 40" and the ordinary light flint glass "O 376" respectively. Using the values of the refractive indices given in the table above we see from equation (1) that we must have $\frac{60.9}{1/f_1} = \frac{42.9}{1/f_2}$. The focal power of the combination, $\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2}$. The focal powers of the combination and of the component lenses must therefore be proportional to 18, 60.9, and 42.9 respectively. If we halve these numbers we obtain the answer to the question proposed. The power of the convergent lens of crown glass is 30.45 and its focal length $\frac{1}{30.45}$ meter, the focal length of the divergent lens of crown glass is $\frac{1}{21.45}$ meter, and the focal length of the combination is $\frac{1}{9}$ meter.

It will be seen on glancing at the table above that all the flint glasses have a greater mean dispersion than the crown glasses and in general a greater refractive index. The latter is not always the case however, for the dense barium phosphate crown glass has a high refractive index 1.576 combined with low mean dispersion 0.00884. It is necessary to employ a crown glass with properties similar to this for the convergent lens, if an achromat capable of giving a flat field is to be constructed.

It is possible to lessen greatly the spherical aberration of objectives by putting in stops, but this lessens the amount of light and so lengthens the time of exposure. The best lenses are therefore constructed from calculations (on the radii of curvature, refractive indices, and the distances between the lenses), with a view to eliminating spherical aberration and producing a flat field without the use of a narrow stop. As an instance of a very successful attempt at this we may mention Petzval's portrait objective.

Students wishing to perform experiments in photography will find directions for the use of the apparatus in this laboratory in Minor's *Physical Measurements,* pp. 87-92.

QUESTIONS.

1. Show how a telescope could be made by the use of two mirrors, and a convergent lens (for the eye-piece)?

2. A grating having 500 lines to the centimeter is placed on a spectrometer table. At what angles with the direct position of the observing telescope will the first, third, and fifth diffracted images in sodium light be found?

IV. POLARIZED LIGHT.

Experiment 30. Plane-Polarized Light by Reflection. Measurement of Angle of Complete Polarization. Double Refraction in Calcite. (*a*) Take a piece of photographic plate glass that has a darkened film on one surface. Obtain a reflection of a sodium flame, the air-glass surface being turned towards the flame. Call this image of the flame I_1. Are you able to extinguish it by varying the angle of incidence of the light?

Set up the apparatus as indicated in figure 25 so that an image I_2 of the image I_1 may be obtained by the use of a second plate of glass M_2. A broad sodium flame (obtained by allowing the flame of a fish-tail Bunsen burner to play against an asbestos edge with common salt on it) is placed behind the opening O in the screen. The mirrors M_1 and M_2 are mounted on arms by means of which they are capable of rotation about a horizontal axis. When the plane of incidence and reflection of the light observed by the eye from the mirror M_2 is at right angles to the plane of incidence and reflection of this light for the mirror M_1, it will be found possible by varying the angles of incidence to practically

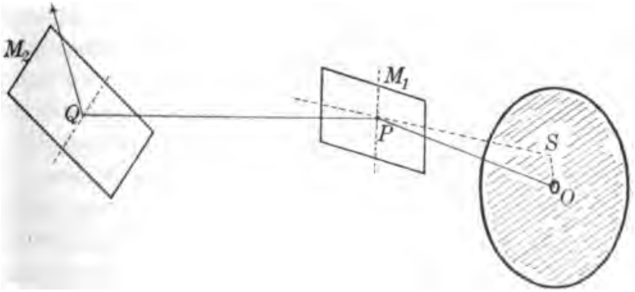


Fig. 25. Illustrates Polariscope of Malus.

extinguish the image I_2. The angles of incidence on both the mirrors should then be equal. Measure the angle of incidence on M_1 as follows. With a piece of pointed chalk laid on the surface M_1 find where the light that goes to form the image I_2 (in the adjustment described above) strikes the mirror M_1. Call this place P. With the aid of two meter rods find the perpendicular distance from O to the plane of the mirror M_1. Let this be OS. Then $\frac{OS}{OP}$ is the cosine of the angle of incidence θ. If this is the angle of incidence on each of the mirrors when I_2 is extinguished, then

$$\tan \theta = \mu_D$$

where μ_D is the index of refraction of the glass for sodium light.

(*b*) Calcite crystals are rhombs, their faces being parallelograms whose sides make angles of 102° and 78° with each other respectively. There are therefore two opposite blunt corners at which the edges form three equal angles of 102° with each other.

Take a large calcite crystal with natural faces and preferably one that has two faces each with four equal edges. Find a direction through two opposite faces of the crystal such that along it only one image of an object is obtained. It will be found that this direction is parallel to a line through either of the blunt corners that makes equal angles with the three edges that terminate in the blunt corner. This direction is called the optic axis. The term is equally well applied to any line through the crystal that is parallel to the line making equal angles with the edges at the blunt corner. If we are observing objects through two parallel faces of the crystal, then any plane perpendicular to the refracting faces and parallel to the direction of the optic axis is called the principal plane.

Draw two long black lines at right angles to each other on a piece of white paper and put a heavy dot at the intersection of the two lines. Put the crystal (with natural faces) on the paper on the table. Look straight down at the black point. By rotating the crystal and noticing the directions of the long lines on the paper outside the crystal it will be seen that the moving point (known as the extraordinary image) always appears to lie in the same principal plane as the ordinary image. The principal plane through the point on the card is therefore the plane which contains the two images of the point when a line joining the eye to the point is perpendicular to the two faces. Figure 26 shows the course of the ordinary and extraordinary rays in the principal plane of the crystal. O is the point we have been considering. OE is the direction of the optic axis, and the principal plane is therefore the plane of the paper and must contain both the ordinary and extraordinary rays from O that come out of the crystal perpendicularly to the face DC. The circle and ellipse both centered at O show a section of the wave-fronts of the light from O, the wave-front for the ordinary rays being a sphere and that of the extraordinary rays an ellipsoid. It may be seen from the course of the extraordinary rays that come to the eye, namely along OTM, that the extraordinary rays do not obey the law of refraction that we have become familiar with in studying isotropic substances. The ordinary ray however does obey this law.

Place on the table a pile of glass plates. Make a small opening in a piece of paper and look perpendicularly through the opposite faces of the crystal and the small opening in the paper toward the pile of plates. Light reflected from the pile of plates near the polarizing angle will give two images of the opening which are in general of different intensity. Turn the crystal around. Defining, as is the convention, the plane of polarization of the light from the plates as the plane of incidence on (and of reflection from the plates), we see that the ordinary image is bright when the principal plane of the crystal is parallel to the plane of polarization, that is the plane containing the two images is parallel to the principal plane. The ordinary ray is therefore made up of light polarized in the principal plane. If the crystal be now turned until a line joining the two images appears to be at right angles to the plane of reflection from the pile of plates, the extraordinary image becomes the brighter. The extraordinary ray therefore consists of light polarized at right angles to the principal plane.

It will be evident from the previous study that if the refracting surface of the natural calcite crystal be a parallelogram with equal sides, then a principal plane of the crystal must be the plane perpendicular to the refracting surface and containing the diagonal through the blunt corner.

Examine the model of a Nicol prism. Also examine the light from a pile of plates by means of a Nicol prism. It will be found that the light that comes through the Nicol is polarized at right angles to the principal plane.

(*c*) Mount a Nicol prism in a tube which rotates about an axis on a horizontal table. This is a convenient arrangement for measuring the angle of maximum polarization of various surfaces that may be placed on the horizontal surface. Do this for crown glass, for heavy flint glass, for water, for polished brass.

Experiment 31. Study of Circularly Polarized Light and Elliptically Polarized Light. With the arrangement indicated in figure 27 light from the sodium flame is partially reflected downward by a glass plate P at about the polarizing angle. The light that is so reflected is plane-polarized in the plane of reflection. It is reflected nearly normally upward by the surface M through the plate P to the Nicol prism N.

Rotate the Nicol until the light that comes up from the surface M is extinguished. Place on the surface M a thin plate of mica. Rotate the mica plate on the surface M. Two positions of the mica plate will be found for which it appears dark. Mark these positions by means of pencil marks (on the mica) that are parallel and perpendicular respectively to the plane of polarization of the incident light. Turn the mica through an angle of 45° from one of these positions. The incident polarized light is now broken up into two equal components, one of them polarized in a plane through the optic axis, that is the principal plane, and the other polarized at right angles to the principal plane. These two components travel through the mica with different velocities. If the path through the mica is sufficiently long to produce 180° difference of phase between the two components they will combine after leaving the mica into a resultant which is polarized

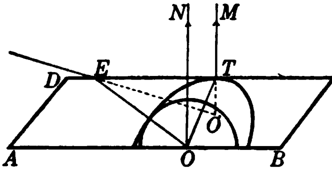


Fig. 26

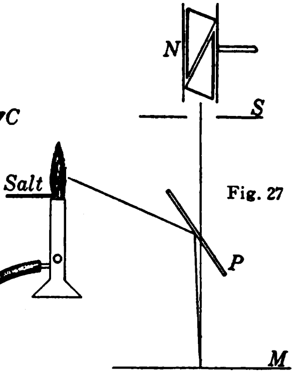


Fig. 27

Fig. 26. Course of Ordinary and Extraordinary Rays in the Principal Plane of a Calcite Crystal.

Fig. 27. Apparatus for Obtaining Circularly and Elliptically Polarized Light.

at right angles to its previous plane of polarization. In this case if the Nicol be now turned through a right angle, the mica will appear black while the glass plate beside it appears bright. The student therefore is asked to test the pieces of mica furnished him to see whether he can find any which appear black through the Nicol while the portions of the plate M not covered by the mica are of their maximum brightness. If such a piece of mica be now moved from M up to S, keeping the directions of the pencil marks on the mica parallel to their directions when on the lower surface, he will find that the mica appears equally bright for all positions of the analyzing Nicol as the latter is rotated. The light that comes through the mica now is circularly polarized light. If the mica be rotated the light that comes through it will be in general elliptically polarized light, though there are two positions where it will be plane-polarized light and two positions where it will be circularly polarized. The explanation of this phenomenon is that the light passes through only half the thickness of mica at S as compared with the thickness of mica traversed at M. If the difference of phase introduced between the two equal components at M is 180°, the difference of phase introduced at S is only 90°. But two equal vibrations in planes at right angles form a circular vibration when the difference of phase between them is 90°. Such a plate, prepared in the way we have described is called a quarter-wave plate.

Label the quarter-wave plate and mount it suitably in your notebook.

Experiment 32. Rotation of the Plane of Polarization by Means of Quartz Crystals Cut Perpendicularly to the Axis. The quartz crystals are placed in a trough between two crossed Nicols. Light from a sodium burner enters one of these which may be called the polarizer, then traverses the quartz crystal in a direction parallel to its axis, and then passes through the analyzer. The analyzer is provided with an index and vernier by means of which the angle, through which the analyzer has to be rotated from its zero position (in order to extinguish the light that comes through the quartz crystal), may be read from an angular scale. The zero position of the analyzer is obtained by removing the quartz and turning the analyzer again to the position of extinction. There are always however two positions of extinction 180° apart. Some quartz moreover rotates to the right and some to the left according to the molecular structure of the crystal. Right-handed quartz may be distinguished from left-handed quartz by using different sources of monochromatic light during the experiment. This follows from one of Biot's laws. This law states that the rotation for a given thickness is inversely as the square of the wave-length.

Measure the rotation of the plane of polarization produced by a given quartz crystal, when the light traverses it in a direction parallel to the axis, for sodium yellow and for lithium red. The wave-lengths of these kinds of light in air are in millionths of a millimeter 589 and 671 respectively. The light transmitted by a solution of ammonium cuprate in water may also be tried. This gives a roughly monochromatic blue light.

Compare the rotations observed and see whether the ratios between them are what you would expect from Biot's law.

Repeat the experiment with other similar quartz crystals of different thicknesses and directions of rotation.

Calculate from all your data the mean rotations per millimeter length of path in the quartz for the three colors described above.

Saccharimetry. This is an exceedingly practical application of polarized light. The rotation produced in a water solution of cane sugar by sodium light is proportional to the length of the path traversed in the solution and to the concentration of the solution. There is a Laurent's saccharimeter in the laboratory. Students wishing to perform the experiment will find full directions in Kohlrausch's *Physical Measurements.*

QUESTIONS.

1. Describe two methods by which the velocity of light in air has been determined.

2. Explain the advantages of oil immersion in microscopy.

3. How can you determine the position of the optic axis in a natural calcite crystal?

4. What is the cause of the Fraunhofer lines? How can you distinguish terrestrial lines from solar lines?

5. For a given star at a given time the C line ($\lambda = 6563$ Ångström units for sources at rest with respect to the observer) is found to be displaced one Ångström unit toward the violet. Is the star coming toward the earth or going away from it and how fast in the line of sight?

6. Compare the angles of maximum polarization for Jena glasses O 225 and S 57 (see table on page 100).

7. What would be the focal lengths of the convergent and divergent lenses made of O 225 and S 57 respectively in order that the combination should be achromatic and have a focal length of 100 centimeters?

8. Calculate the rotations of quartz cut perpendicularly to the axis for light of the C, F, and H lines respectively, the rotation for sodium light being 21.7° per millimeter. The wave-lengths in air for the above kinds of light are 0.656, 0.486, 0.394, and 0.589 thousandths of a millimeter respectively.

ELECTRICITY AND MAGNETISM

I. FUNDAMENTAL EXPERIMENTS IN ELECTROSTATICS.

Quantity, Potential, Capacity. If two similar pith-balls each of mass m be suspended from a common point (by two equally long fine silk fibres) and equally charged by contact with a third body, the two balls will take up positions of equilibrium if there be no conduction of electricity from them. Let the distance between the centers under these conditions be r and the angle between the suspending fibres be 2α. Let the charge on each pith-ball be e and suppose the distance between the centers of the balls to be large as compared with the dimensions of either ball. Then the force acting between the charges is $\frac{e.e}{r^2}$, if the surrounding medium is air. The force on either of the balls is balanced by the horizontal component of the tension of the fibre, that is $\mathrm{T}.\sin\alpha$, where T is the tension in the fibre expressed in dynes. Therefore

$$\mathrm{T}\sin\alpha = \frac{e^2}{r^2},$$

and

$$\mathrm{T}\cos\alpha = m\,g,$$

since the vertical component of the tension of the string is balanced by the weight it supports. Dividing one equation by the other we obtain

$$\tan\alpha = \frac{e^2}{r^2 m\,g} \text{ and } e = r\sqrt{m\,g\tan\alpha}.$$

Therefore the quantity e of electricity on either pith-ball may be calculated if the distance between the balls r, the angle between the suspending fibres $2a$, and the mass of each pith-ball, m, be measured. Such measurements of quantity of electricity as that described above would be necessarily rough because the charges on the two balls would not be uniformly distributed over their surfaces owing to the repulsion between the two like charges.

The potential of a charged body is defined as the amount of work that would have to be performed to bring unit quantity of electricity from a place infinitely distant from charged bodies up to the surface of the given charged body. If the latter be for example a sphere of radius R situated away from other charged bodies and having a charge E uniformly distributed over its surface, then the force exerted on unit charge at a point distant r from the center of the sphere is $\frac{E.1}{r_2}$. To calculate the amount of work done in bringing up unit quantity requires a summing up for each element of the path of the product of the force into the distance gone against the force. This sum taken from an infinite distance to any point distant r from the center of the sphere is $\frac{E}{r}$. The potential at the surface of the sphere is therefore $\frac{E}{R}$. The equipotential surfaces outside the large sphere are spheres concentric with it. Inside the charged sphere the potential is uniform and equal to $\frac{E}{R}$ if there are no charged bodies inside.

The capacity of a charged body is equal to the quantity of charge on it divided by its potential. The capacity of the charged sphere described above is then $\frac{E}{\frac{E}{R}} = R$. The capacity of an isolated sphere is therefore equal in electrostatic units to its radius. The capacity of the sphere would be very different if for example there were near it another sphere charged with the same kind of electricity. For a given charge on the original sphere it would in that case require more work to bring up unit charge from an infinite distance. The expression quantity divided by potential would therefore be less than before, that is the capacity of the original sphere is diminished by bringing up another with a charge of the same sign. On the contrary its capacity would be increased by bringing up a sphere with an opposite charge. Such an arrangement would form a condenser. Condensers are usually made of tin foil separated by glass, paraffined paper, or mica. By taking large surfaces of tin foil separated by thin sheets of the insulating material it is possible to collect very large quantities of opposite kinds of electricity on the opposite surfaces of the insulator with only a small difference of potential. Such an arrangement therefore gives a condenser of very large capacity.

Experiment 33. Use of the Gold-Leaf Electroscope. This instrument ordinarily has two strips of gold leaf suspended from a metal rod, the leaves and the metal rod being carefully insulated from the surroundings. This insulated system is placed inside a metal box provided with windows for observing the deflection of the leaves when charged. The metal box is connected with the earth to keep it at constant potential. The leaves are charged by means of a wire that runs in through a cork in the metal box and may be disconnected when desired. Calcium carbonate may be kept inside the box to keep the atmosphere dry.

(*a*) The gold leaves may be charged by direct contact or by induction. With a piece of sealing wax that has been rubbed on the coat sleeve a small charge may be given to the leaves through the charging wire. The charge thus given is according to the present convention negative. A positive charge may be given to the leaves by contact through the charging wire with smooth glass that has been rubbed with dry silk. Test the sign of the charge by noticing the inductive effect when the charged sealing wax is brought near without charging by direct contact. By the decrease or increase of the deflection of the leaves, when the sealing wax is brought near, the sign of the charge on the leaves is easily determined.

To charge the leaves by induction first discharge them by connecting them to the earth through the hand when there is no charged body near. With the other hand bring up the charged sealing wax, then break the connection of the leaves to the earth, lastly remove the charged sealing wax. The leaves are now found to be charged positively. The leaves might similarly be charged negatively by induction if a glass rod that had been rubbed with silk were used instead of the sealing wax.

(*b*) **Causes of Leakage.** The gradual diminution of the deflection of the gold leaves may be due to many causes. A film of moisture over the surface of the insulators is a frequent cause. Air itself however may be made conducting by very short waves of light, by Röntgen rays, by radiations from radium, etc. To illustrate this put the connecting wire in electrical contact with the leaves. Light a match near the outer terminal. Is there any change in the rate of discharge of the leaves? Obtain a spark between aluminum or zinc terminals of an induction coil. Focus the light near the outer terminal of the gold leaves by means of a concave mirror. See whether there is any difference in the rate of discharge of the gold leaves according as they are charged positively or negatively. Attach to the outer terminal of the wire a fine-pointed needle. Does this make any difference in the rate of leakage from the leaves?

(*c*) **Distribution of Charge on a Conductor.** Charge a large egg-shaped conductor from an electrophorus or an influence machine. Discharge the electroscope. By means of a small proof sphere on an insulating handle carry charges from various parts of the large conductor to the electroscope and note the deflections of the leaves (from position of zero charge) in the various cases. Make a drawing of the large conductor and record the various points tested around a given section showing the magnitude of the corresponding deflection of the gold leaves.

(*d*) **Distribution of Potential around a Charged Conductor.** A small proof sphere is connected by means of a very fine copper wire to the outer terminal of the electroscope. This is now moved around a charged conductor without making contact. The deflection of the gold leaves now gives an indication of the distribution of potential around the charged conductor, if the induction on the small wire can be neglected. In that case the equipotential surfaces can be roughly mapped out by noticing the positions of the proof sphere for which the deflections of the gold leaves are equal. Test in this way the distribution of potential around two similar large spheres equally charged. They may be charged while in contact and then separated a few centimeters. The charges will divide equally if the spheres are equal. Repeat the experiment with two spheres charged oppositely, one sphere being charged by induction from the other, when there is only a small gap between them.

Experiment 34. Calibration of an Electroscope. Measurement of Capacity. When the electroscope is provided with a scale as in figure 28 and calibrated so that the instrument may be used to measure potential in volts, it becomes an electrometer.

For measurements of large differences of potential it is well to use an aluminum leaf about 60 mm. long and 2 to

6 mm. wide. This is repelled by a fixed brass plate as shown in figure 28, when the leaf A and its metallic connections are charged. The deflections are read by means of an angular scale backed by a mirror. The half-cylindrical outer wall of the instrument is kept connected with the earth so that it is constantly at zero potential. It forms with the leaf and the connections of the latter a condenser whose capacity is to be determined later. The deflection of the aluminum leaf depends on the difference of potential between the leaf and the walls of the electrometer, on the position of the latter with respect to the insulated system, and also on the weight of the leaf. It is found that the deflection α corresponding to the potential V is approximately given by the equation

$$V = k\sqrt{m.g}\sqrt{\sin\alpha \cdot \tan\alpha},$$

where k is a constant for the given instrument and $m\ g$ is the weight in dynes of the leaf. The leaf is connected (by means of the brass plate and a brass rod that runs through a piece of hard rubber in an opening in the wall to a hollow pail F. This is used as in Faraday's ice-pail experiment to completely discharge any charged body brought in contact with the inside of the pail.

(*a*) Make a relative calibration of the instrument by adding successive equal quantities of electricity to the insulated system, commencing with it in an uncharged condition. Charge the carrier from the inner conductor of a very large charged Leyden jar whose outer coating is connected to the earth. Keep the Leyden jar about two meters from the electrometer and protect the latter by means of a wire cage with earth connection. The carrier may be a sphere two centimeters in diameter mounted on an insulating rod not less than 50 cm. long. The Leyden jar should be sufficiently charged to give a deflection of from 8-12 degrees for the first charge. Note the deflections obtained by suc-

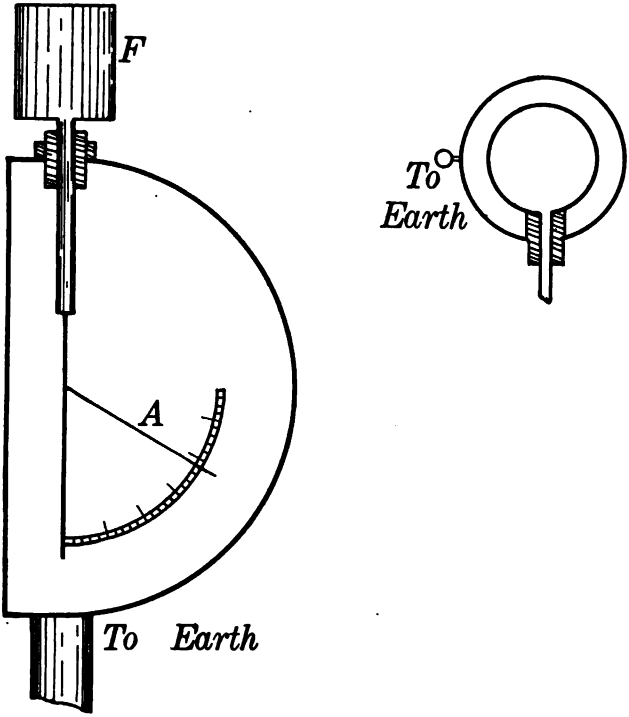


Fig. 28. Electrometer for Simple Capacity Measurements. Spherical Condenser.

cessive additions of equal quantities until a deflection of from 70° to 80° is obtained. Discharge the electrometer and obtain a deflection corresponding to the first addition of charge to F. If the same value is obtained again the results may be plotted. If not it indicates that the Leyden jar has been leaking and another jar should be used.

Assuming that the capacity of the insulated system does not depend appreciably on the position of the leaf, the potentials of the leaf and its connections are proportional to the quantity of charge on the insulated system. The results obtained above may then be expressed as a relation between the potential corresponding to any deflection and the potential that gives a deflection of 10° for example. The calibration is thus far simply relative.

Connect one terminal of a hundred storage cells arranged in series to the wall of the instrument and connect the other terminal to the hollow pail F. These cells give a potential of about 200 volts to the insulated system. The difference of potential at the terminals of the cells may be measured more exactly by means of a voltmeter applied to a suitable number of cells in series. If the cells are properly insulated the difference of potential at the terminals is equal to the sum of the difference of potentials obtained for the various sections of the battery. Note the deflection of the electrometer for the difference of potential at the terminals of the battery. Other deflections (included within the relative calibration) may now be expressed in volts. Make a curve in which the abscissæ are deflections of the leaf and the ordinates are volts.

(*b*) Calculate the capacity of the spherical condenser furnished (using the formula for a spherical air condenser whose outside metal shell is connected to the earth) from $C = \frac{r.R}{R - r}$, where r is the radius of the inner condenser surface, and R is the outer radius of the enclosed shell of air. Charge the insulated system of the electrometer until a deflection of 50° to 60° is obtained. Let the potential corresponding to this deflection be V. Let the capacity of the electrometer system be C_e. Then the charge on the insulated system is $Q = C_e . V$. Connect the insulated system by means of a very fine wire to the inner surface of the spherical condenser. The charge will now divide between the two systems and the potential of the electrometer and the inner metal shell will be some value v indicated by the new deflection of electrometer. Then since the whole quantity of charge is the same as before

$$C_eV = C_e \,.\, v + \left(\frac{r.R}{R - r}\right) v.$$

Therefore the capacity of the electrometer is

$$C_e = \frac{r.R \times v}{(R - r)\ (V - v)}$$

Calculate the capacity of the electrometer furnished.

(*c*) To measure the specific inductive capacity of the paraffin in the laboratory, fill a spherical condenser (similar to the one used in part *b*) with paraffin. Charge the electrometer alone to some potential V, then divide the charge between the two condensers. Let the resultant potential be v. Then since the total charge remains the same

$$C_eV = C_ev + \frac{K(r.R)\,v}{(R - r)},$$

from which the specific inductive capacity K may be calculated if the radii of the condenser surfaces R and r have been measured.

Report the specific inductive capacity of the paraffin.

Experiment 35. Measurement of Spark Potentials with the Bichat-Blondlot Absolute Electrometer. An absolute electrometer is one that gives indications from which the potential may be calculated by means of a simple formula derived from the form of the instrument. In the case of the instrument used in this experiment (see figure 29) a light aluminum cylinder is supported on the end of a balance arm and is connected to earth. The aluminum cylinder projects into a larger cylinder that is carefully insulated and is at potential V. When the insulated cylinder is charged the aluminum cylinder is attracted with a force which may be balanced by adding weights to the pan

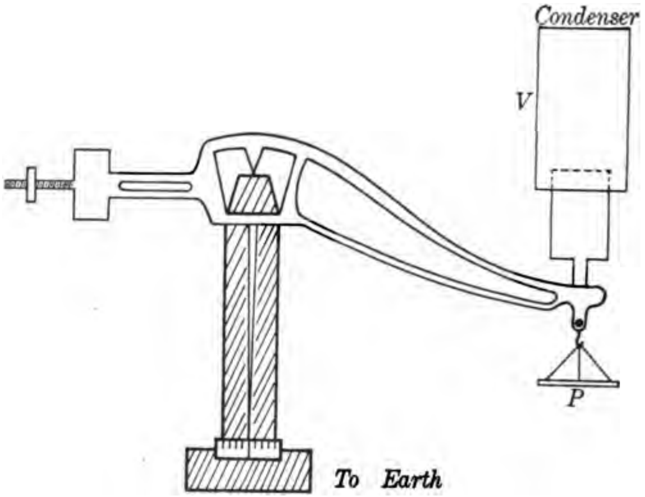


Fig. 29. Blondlot-Bichat Absolute Electrometer.

P of the balance so that the zero position of the pointer is maintained. The capacity of a cylindrical air condenser is equal to $\frac{L}{2\log_e \frac{b}{a}}$ where L is the length of the smaller cylinder enclosed by the larger and b and a the radii of the two condenser surfaces, b being the larger. The energy of a charged condenser is equal to $\frac{1}{2}.C\,V^2$. The force that attracts the smaller cylinder is equal to the rate of increase of energy as L increases. This is $\frac{1\;.\;V^2}{2\left(2\log_e \frac{b}{a}\right)}$. If m grams are necessary on the balance pan to bring the pointer to the zero when the condenser is charged, then we have

$$m\,g = \frac{V^2}{4 \log_e \frac{b}{a}}$$

As an example of the use of this instrument sparks may be obtained between two zinc balls each about one centimeter in radius by means of an influence machine. One of the terminals of the spark gap is connected to the earth, the other is connected to the insulated cylinder of the electrometer. The latter is surrounded by a wire cage not shown in the figure. The circular glass plate of the influence machine is turned slowly by one student while the other adds weights to the pan of the balance. The maximum weight required for equilibrium just before the spark passes is noted. Determine the spark potential for a number of different distances between the zinc terminals.

QUESTIONS.

1. A conducting sphere (of 30 cm. radius) remote from other charged bodies is charged to a potential of 1000 C. G. S. units. What is the quantity of electricity on its surface? What is the potential at a point 50 centimeters from the center of the sphere, 10 centimeters from the center?

2. How much work will be required to charge the sphere described in question 1 to a potential of 500 C. G. S. units?

3. Calculate the capacity of a spherical air condenser whose outside and inside radii are 30 and 32 centimeters respectively, the outside being connected to earth. Express the result in centimeters. What would be its capacity if filled with paraffin?

4. What is the source of the energy of the spark obtained from an electrophorus cover?

5. Explain the workings of a Toepler-Holtz machine.

6. Two very small spheres each charged with 100 C. G. S. units are situated 80 centimeters apart. Plot a section of the equipotential surface V = 5 C. G. S. units using the equation $\frac{100}{v_1} + \frac{100}{v_2} = 5$, where v_1 and v_2 are measured from the respective charged bodies.

II. MEASUREMENT OF MAGNETIC MOMENT AND OF FIELD STRENGTH.

The definition of magnetic pole is quite analogous to the definition of unit electric charge. While the electric charge however is found on the surface of the conductor of electricity, the interior of an iron magnet is found to be magnetized throughout. The effect of a long bar magnet on outside points at some distance from the magnet may be approximately calculated if we know the distance of the outside points from two places called the poles of the magnet and if we know the strength of the poles. In a cylindrical bar magnet that has been carefully magnetized in a solenoid we may take the distance between the poles as being nearly equal to five sixths of the length of the cylinder. Each pole in this case is therefore considered to be one twelfth (of the length of the magnet) from its end. As the position of the poles is difficult to determine directly measurements should be made of the field strength due to the magnets at a considerable distance from either pole.

Experiment 36. Determination of the Ratio of the Magnetic Moment of a Magnet to the Intensity of the Horizontal Component of the Earth's Field.. The bar magnet is placed in this experiment so that its axis is approximately perpendicular to the magnetic meridian. Let the pole-strengths be denoted by $+m$ and $-m$ respectively. Let the magnetic length (taken as five sixths of the length of the bar) be $2l$. At a distance L measured along the axis of the magnet (starting from its center) a compass needle is mounted. Let its magnetic length and its pole-strengths be represented by 2λ, $+\mu$, and $-\mu$ respectively. Let the horizontal component of the earth's intensity at the center of the compass needle be H. The compass needle will take up some position in which it makes an angle θ (measured in a horizontal plane) from the position of the needle when the bar magnet is removed a great distance. When the center of the bar magnet is distant L centimeters in the position described above, the north pole of the compass needle is acted on by the forces $\frac{m.\mu}{(L-l)^2}$ and $\frac{m.\mu}{(L+l)^2}$ due to the two poles of the bar magnet respectively. If the compass needle is short as compared with the distance from either pole of the bar magnet, these two forces are so nearly parallel that we may obtain their resultant by subtracting one from the other. The force on the south pole of the compass needle will be equal numerically to that on the north pole but opposite in sign. Here again an approximation is made because in reality one pole of the compass needle will be a little farther from the bar magnet than the other. The perpendicular distance between the two forces at the compass needle is $2\lambda \cos\theta$. The moment of the couple exerted by the bar magnet on the compass needle is approximately

$$\left\{\frac{m\,\mu}{(L-l)^2}-\frac{m\,\mu}{(L+l)^2}\right\}2\lambda\cos\theta.$$

The couple exerted by the earth's field on the compass needle is

$$H\,\mu.2\lambda\sin\theta.$$

For equilibrium these two couples must be equal. Therefore equating we get

$$\frac{M}{H}=\frac{2\,m\,l}{H}=\frac{(L^2-l^2)^2}{2\,L}\tan\theta,$$

where M is the magnetic moment of the magnet. In this and in the succeeding experiments in magnetism great care should be taken not to change the magnetic moment of the magnet. It should be handled with a cloth in order to avoid great temperature changes and it should not be subjected to mechanical jars.

A bar magnet about 30 centimeters long and 5 millimeters in diameter may be conveniently mounted on a carriage that slides on a meter rod (that has been attached to a stiff board). In order to make observations for which the value of L is large it is better to have two meter rods on the board in line with each other. The board is provided with leveling-screws so that the surface on which the carriage slides may be in a horizontal plane. With the aid of a long compass needle and a T-square the meter rods may be made perpendicular to the magnetic meridian at the place where the two rods join, which we shall call the center of the scale. The long needle is now removed and a compass-box (with a scale in degrees and a short needle with a long pointer) is placed over the scale in such a way that the center of the angular scale and the point of support of the short needle are just over the center of the linear scale. The magnet is placed in its carriage so that its axis is parallel to the direction of motion of the carriage and so that its axis is at the same height as the compass needle. For a given value of L four readings may now be taken for the deflection of the compass needle, two by reversing the magnet in its carriage on the east side of the compass needle and similarly two for the west side. Each reading of the deflection of the needle should however be obtained by taking the mean of the readings for the ends of the needle approximately 180° apart. In this experiment and in all succeeding magnetic measurements it is desirable to remove magnetic material such as keys and knives to some distance from the magnetic needle.

A set of data for a given distance L between the centers of the two magnets may be conveniently taken as indicated below.

BAR MAGNET EAST OF COMPASS.		BAR MAGNET WEST OF COMPASS.	
No. Pole E.	No. Pole W.	No. Pole E.	No. Pole W.
$L = 40.1$ cm.	$L = 40.0$ cm.	$L = 39.9$ cm.	$L = 40.0$ cm.
$\theta = 15.1°$	$\theta = 14.9°$	$\theta = 15°$	$\theta = 15.2°$

In calculating tabulations may be made as follows:

	$L =$	$L =$	$L =$
θ (mean of four)	......	.....	......
L (mean of four)	......	......	......
l	......	......	......
log. $(L + l)$	......	.....	......
log. $(L - l)$	......	......	......
log. $(L^2 - l^2)^2$	......	......	......
colog. 2L	......	......	......
log. tan θ	......	......	......
log. $\frac{M}{H}$	......	......	......
$\frac{M}{H}$	......	......	......

Repeat the experiment with different values of L extending from 30 centimeters to 80 centimeters and calculate the value of $\frac{M}{H}$ for each case.

Average the values obtained for $\frac{M}{H}$. Calculate the greatest percentage difference from the mean value so as to form an idea of the possible accuracy of your work apart from constant errors in scales, etc.

Experiment 37. Determination of the Product M H. Bifilar Suspension. The same bar magnet that was used in the last experiment is suspended by means of a bifilar suspension as indicated in figure 30. In addition to the magnet however the wires have to support a carriage and mirror. If the load is symmetrical with respect to the two strings (each of length l), each string has to sustain the weight $\frac{mg}{2}$, where m is the mass of the magnet, carriage, and mirror. When the strings are vertical the magnet is perpendicular to the magnetic meridian. This position is denoted by NDFS in the figure. If the magnet be released

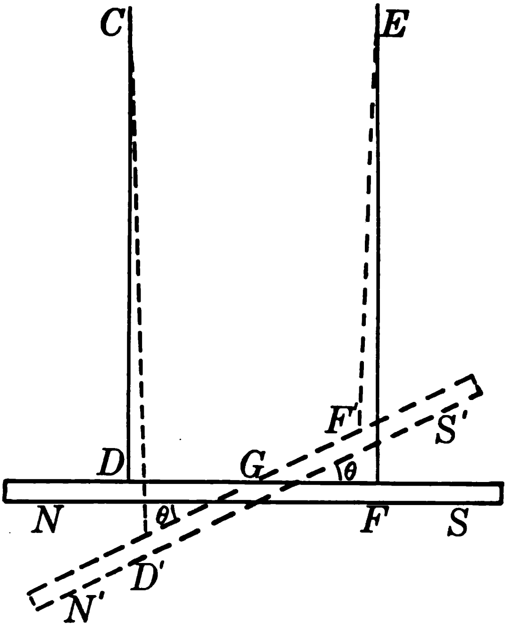


Fig. 30. Bifilar Suspension for Magnet.

it swings into the position denoted by the dotted lines in the figure. Let a be the distance between the suspending threads in their original parallel position. The foot of each thread has now moved through the distance DD′ $= \frac{a}{2} \cdot \theta$, where θ is the angle of deflection of the magnet from its original east and west position measured in radians. The force exerted by the string CD on the magnet is directed along D′D and is equal to the horizontal component of the tension of the string. The tension is $\frac{m\,g}{2}$. The sides of the triangle DD′C represent the tension of the string and its horizontal and vertical components in magnitude and direction. Therefore the horizontal component is equal to

$$\frac{\mathrm{DD}'}{\mathrm{CD}} \cdot \frac{mg}{2} = \frac{a \,.\, \sin\theta}{2\,(\mathrm{CD})} \cdot \frac{mg}{2},$$

since the angle θ is small.

The couple exerted in a horizontal plane by the strings is obtained by multiplying this force by the perpendicular distance between the two forces, namely a. The restoring couple is therefore

$$\frac{a^2 .}{2\,(\mathrm{CD})} \sin\theta . \frac{mg}{2}.$$

The horizontal component of the magnetic couple is $\mathrm{H}m . 2\,l \cos\theta = \mathrm{MH}\cos\theta$, where $2\,l = \mathrm{NS}$. For equilibrium these two couples must be equal. Therefore

$$\mathrm{MH} = \frac{a^2 \; m\,g}{4\,(\mathrm{CD})} \cdot \tan\theta.$$

The magnet used in the last experiment should be suspended near the position occupied by the compass needle in the previous experiment, that is the position for which $\frac{\mathrm{M}}{\mathrm{H}}$ was determined. To get the plane of the wires (when they are in a vertical plane at right angles to the magnetic meridian) a brass rod is placed in a carriage (used later to hold the magnet NS) and the wires adjusted approximately with the aid of a compass needle. The errors due to inexact adjustment in this respect may be nearly eliminated later by reversing the magnet in its carriage and taking the mean of the two values obtained for θ. The readings of this angle are taken by means of a telescope and scale. The scale is adjusted so that the distance from the center of the mirror to either end of the scale is the same. This may be tested conveniently with a stretched string. The axis of the telescope is set over the middle point on the scale and an image of this point is obtained at the intersection of the cross-hairs in the telescope (when the brass bar is mounted in the carriage). To protect from drafts the carriage and bar are screened with a box which has a slit above (for the long suspending wires to pass through), while there is a window in front for the purpose of reading the deflection of the magnet. To dampen the vibrations a paddle is suspended from the carriage into water or oil.

After the initial adjustments have been made, place the magnet in its carriage with the north pole toward the east, for example, and obtain the scale reading (when the oscillations have become of small amplitude) by averaging the successive readings obtained at the end of a swing. One more reading should be taken on one side of the equilibrium position than on the other. Reverse the magnet in its carriage so that its north pole is now toward the west, and obtain again the scale reading. Repeat a number of times. The mean scale distance is one half of the difference between the scale readings obtained for the two opposite positions of the magnet, each of these positions being determined by a number of readings as suggested above.

The mean scale distance divided by the distance from the middle point of the scale to the plane of the mirror is equal to tan 2θ, where θ is the angle of deflection of the magnet from an east and west position.

The distance between the wires should be measured at both the upper and lower ends a number of times. The average of the values obtained at the two ends is *a* in the above formula. The data may be taken conveniently as follows:

SCALE READINGS

	NORTH POLE OF MAGNET TOWARD EAST	NORTH POLE WEST
1	......	
2		......
3	......	
etc.		
	Mean......	Mean......

Difference between mean east reading and mean west reading = ...
Mean scale distance = ½ above difference =.............. ......
Distance from middle point of scale to plane of mirror = ...
Tangent 2 θ = ... 2 θ ...... θ = ...
Distance between wires at upper end, mean of .. readings, = ...; distance at lower end = ...
a, mean of above distances, = ...
Length of wires, CD = ...
Mass supported by wires, *m* = ...

The calculations may be conveniently tabulated as follows:

Log a^2 = ...
Log *m* = ...
Log *g* = ...
Log tan θ = ...
Colog 4 $\overline{CD}$ = ...
$\overline{\text{Log MH}}$ = ...
MH = ...

The values obtained for $\frac{M}{H}$ and MH in the last two experiments for the same magnet and for the same position may be combined so that both M and H may be determined. Define carefully the position at which you have determined H, *x* meters from south wall, *y* meters from east wall, *z* meters from floor.

The intensity of magnetization of the magnet may now be obtained by dividing the magnetic moment by the volume. Report the magnetic moment and intensity of magnetization of the magnet as well as the number of the magnet on the attached label.

Experiment 38. Comparison of the Values of H in Different Parts of the Laboratory. (*a*) **The Variometer.** The method given below is not the best for detecting variations in H, but is the simplest. The student will find a much better method for detecting small variations in H described in Kohlrausch's *Physical Measurements.* The values of H may however be roughly compared by mounting a permanent magnet at a definite height over a compass needle, the axis of the permanent magnet being as nearly as possible at right angles to the magnetic meridian. If the latter adjustment were exactly made we should have as in experiment 36,

$$\frac{\text{Horizontal Component of Earth's Field at Center of Needle}}{\text{Permanent Magnet's Field at Same Point}} = \text{Cotangent Angle of Deflection of Needle}$$

We may write this in symbols for a given position of the instrument

$$\frac{H}{K} = \text{cotan } \theta. \quad (1)$$

At some other position the equation will become $\frac{H'}{K} = \text{cotan } \theta'$. Since K is a constant if the magnet is carefully moved and not exposed to violent changes of temperature

$$\frac{H'}{H} = \frac{\text{cotan } \theta'}{\text{cotan } \theta}. \quad (2)$$

The errors due to failure in placing the magnet perpendicular to the magnetic meridian may be largely eliminated by reversing the permanent magnet at each of the positions for which H is to be compared. Let the axis of the permanent magnet make an angle ϕ with the magnetic east and west line. Then instead of equation (1) above we have, when the permanent magnet assists the earth's field, $\frac{H + K \sin \phi}{K \cos \phi} = \text{cotan } \theta_1$ and when the permanent magnet is reversed $\frac{H - K \sin \phi}{K \cos \phi} = \text{cotan } \theta_2$, where θ_1 and θ_2 are the deflections from the zero position of the needle (it has the zero position when under the influence of the earth's field alone). Subtracting the lower equation from the upper we get

$$\tan \phi = \tfrac{1}{2} (\text{cotan } \theta_1 - \text{cotan } \theta_2). \quad (3)$$

Adding the same equations we get

$$\frac{H}{K} = (\text{cotan } \theta_1 + \text{cotan } \theta_2) \cos \phi. \quad (4)$$

Hence it is easy to calculate from (3) the angle ϕ that the axis of the permanent magnet makes with the east and west line. The smaller angle of deflection θ_1 is obtained when the south pole of the permanent magnet lies farther to the north than its other pole. The adjustment of the instrument may now be corrected if desired, or similar measurements may be taken at another place. Corresponding to equations (3) and (4) we get values (at the other place) for ϕ' and H'. Comparing the value of H' with that of H in equation (4) we have

$$\frac{H}{H'} = \frac{(\text{cotan } \theta_1 + \text{cotan } \theta_2) \cos \phi}{(\text{cotan } \theta_1' + \text{cotan } \theta_2') \cos \phi'}, \quad (5)$$

where ϕ' is obtained as before. If adjustments are made so that ϕ and ϕ' are each equal to zero the equation reduces to $\frac{H}{H'} = \frac{\text{cotan } \theta}{\text{cotan } \theta'}$. The comparison may probably be made more quickly by making the adjustments approximately with the eye and calculating from (3) and (5). If θ_1 and θ_2 are equal within a degree or two we may use equation (2) putting for θ the value $\frac{\theta_1 + \theta_2}{2}$, and similarly for θ' the value $\frac{\theta'_1 + \theta'_2}{2}$.

Compare the values of H at two points in the laboratory, one of these points being near an iron register and the other remote from magnetic material. Make a rough sketch of the laboratory and indicate on it the points for which you have compared the values of H.

(*b*) **The Magnetic Pendulum.** A bar magnet is suspended from a point vertically above its center by means of a fine silk fibre, so that the long axis of the magnet lies in a horizontal plane. If the magnet be now given a small angular displacement around a vertical axis through the fibre, the restoring couple will be $MH \sin \theta + u\, \theta$, where M is the magnetic moment of the magnet, and u is the torsional couple per unit angle of displacement (using circular measure) and θ is the angular displacement. The fibre is supposed to be free from torsion when the magnet is in the magnetic meridian. The latter is therefore the position of the magnet when at rest. If displaced through a small angle and then released the magnet will perform simple harmonic oscillations about its position when at rest. At any part of the oscillation the angular acceleration is numerically equal to the restoring couple ($MH \sin \theta + u\, \theta$) divided by the moment of inertia I of the magnet. Since it is a simple harmonic oscillation the angular acceleration must also be numerically equal to $\left(\frac{2\pi}{T}\right)^2$ times the angular displacement, where T is the period of complete oscillation.

Therefore

$$\theta \left(\frac{2\pi}{T}\right)^2 = \frac{(MH \sin\theta + u\,\theta)}{I}$$

For small angles sin θ may be taken equal to θ. In that case

$$\left(\frac{2\pi}{T}\right)^2 = \frac{MH + u}{I} \text{ and } T^2 = \frac{4\pi^2 I}{MH + u} = \frac{4\pi^2}{MH\left(1 + \frac{u}{MH}\right)},$$

where $\frac{u}{MH}$ may be called the coefficient of torsion. If the instrument be moved to another place

$$T'^2 = \frac{4\pi^2}{MH'\left(1 + \frac{u}{MH'}\right)}$$

Therefore the ratio of the values of H in the two places is

$$\frac{H}{H'} = \frac{T'^2}{T^2} \cdot \frac{\left(1 + \frac{u}{MH'}\right)}{\left(1 + \frac{u}{MH}\right)}.$$

If the fibre is so fine that the coefficients of torsion are negligible the values of H are inversely as the squares of the periods of complete oscillation at the two places respectively, that is $\frac{H}{H'} = \frac{T'^2}{T^2}$. If the fibre has no twist when the axis of the magnet is in the magnetic meridian the coefficient of torsion may be easily determined by twisting the rod that supports the fibre through 360° for example. This will turn the magnet through some small angle a, so that the upper end of the fibre moves through the angle $(360 - a)^\circ$ with respect to the lower end. The torsional couple per unit angle therefore bears to the magnetic couple per unit angle the ratio $\frac{a^\circ}{360^\circ - a^\circ}$, which is therefore the coefficient of torsion, that is $\frac{u}{\mathrm{MH}} = \frac{a}{360 - a}$.

Set up a telescope and scale in front of the suspended magnet so that its deflections may be read with the aid of a mirror on the rod to which the fibre is attached at its lower end. Twist the torsion-head at the top through 360° and measure the angle $a°$ through which the axis of the magnet is deflected from its position of rest. Twist the torsion-head back to its original position and 360° beyond. Note the angle $a_1°$ through which the axis of the magnet has been rotated from its position of rest. Then

$$\frac{\frac{1}{2}(a + a_1)}{360 - \frac{1}{2}(a + a_1)}$$

is the coefficient of torsion. If this is less than 0.01 it may be neglected in the work in this laboratory.

Use this instrument in the succeeding experiments to compare the value of H (at a spot where it is desired to use a tangent galvanometer for example) with the value of H at a spot where it has been carefully determined by the assistants.

The Tangent Galvanometer. This instrument is described in Watson (pp. 692-694). The coils are usually placed so that the plane of each winding is in the magnetic meridian. A comparison between the field at the center of the coils and the earth's field at the same point is then made as in some of the previous experiments by means of a compass needle. The ratio of the current's field to the earth's field is equal to the tangent of the angle of deflection of the compass needle, that is

$$\frac{\frac{2\pi nC}{10\,r}}{\mathrm{H}} = \tan\theta \text{ or } C = \frac{10\,r\mathrm{H}\tan\theta}{2\pi n}$$

where C is the current in amperes, r is the mean radius of the turns (all supposed to be approximately of the same radius), and n is the number of turns. The following precautions are desirable if the best use is to be made of the instrument.

Remove masses of iron from the neighborhood of the instrument unless these masses of iron are permanent fixtures of the room. The galvanometer should be mounted as far as possible from these.

Note carefully the place (x meters from the east wall, y meters from the south wall, z meters from the floor) at which the instrument is used.

Raise the pivot until the needle swings freely. After the experiment the pivot should be lowered so that the fine point may not be dulled.

The insulated wires leading to the galvanometer should be twisted together so that the needle may not be affected by currents outside the instrument.

Place the coil approximately in the magnetic meridian with the aid of a plumb line and compass needle. Send a current through the coils so as to obtain a deflection of about 45° from the zero position of the pointer. Reverse the current and again read the deflection. The angle ϕ that the axis of the coil makes with the magnetic meridian may then be calculated from equation (3) of the preceding experiment as in the following example, where θ_1 and θ_2 are the deflections obtained for the two directions of the current respectively.

	No Current.	Current Readings.		Deflections.	
		1	2	1	2
East End of Pointer ..	1.5 S.	47.0 S.	44.9 N.	45.5°	46.4°
West End of Pointer ..	0.2 N.	44.0 N.	45.0 S.	43.8°	45.2°
			Means	44.6°	45.8°

Therefore $\theta_1 = 44.6°$ and $\theta_2 = 45.8°$. The angle that the plane of the coils makes with the magnetic meridian is given by the relation (derived for the variometer)

$$\tan \phi = \frac{1}{2}(\text{cotan } 45.8° - \text{cotan } 44.6°) = -0.0282.$$

Therefore $\phi = 1.5°$.

After such a determination it is easy to get the plane of the coil in the magnetic meridian by turning it (in the proper direction) through the angle ϕ, which may be done with the aid of the compass needle when no current is flowing. In all cases however an attempt to eliminate the zero error should be made by reversing the current and taking one half of the deflection obtained on reversal as the deflection (from the zero point) caused by the current.

The needle should be read with a magnifying glass, care being taken to avoid parallax. To diminish the errors due to friction of the needle against the pivot, the table beside the instrument should be tapped while readings are being taken.

The Copper Voltameter. As this is to be used in the next experiment a short account is here given of the precautions that are desirable if accurate results are to be obtained. The voltameter consists essentially of a vessel containing an acidulated solution of copper sulphate in water, into which two copper electrodes dip. If a steady current of C amperes passes between the electrodes for t seconds, an amount of copper (under certain conditions to be described) $C\ t \times 0.000329$ grams will be deposited on the cathode. The copper voltameter measures therefore the quantity of electricity that passes between the electrodes during the existence of the current. It is only when the current is approximately of constant magnitude that a mean value of the current may be obtained for the time of flow, which time has to be measured carefully.

The Copper Plates. The gain plate should have a surface of about 50 sq. cm. per ampere (never much less than

50 and not over 100). The loss plates should be somewhat smaller than the gain plates so that there will not be an undue tendency for copper to gather on the edges of the gain plate. The gain plate should be suspended between two loss plates, about one half to three fourths of an inch from each of them. The main part of the gain plate should be covered by the liquid so that only the tongue passes through the surface. This precaution is taken because there is a tendency to chemical action at the places where the copper is exposed to both the air and the copper sulphate.

Treatment of Gain Plates before Run. Lay the gain plates on a piece of clean glass and scour them with fine sandpaper rounding off the edges and all sharp corners. Wash off the copper particles and grit with a cloth and running water at the tap. Rinse the plate in water containing 5 percent by volume of strong sulphuric acid. Pour a little alcohol over both sides of the plate, allow this to drip off for a few seconds, then touch cautiously to a Bunsen flame. When the plate is dry weigh to the nearest 0.003 gram.

Treatment of Gain Plates after the Run. Wash off the copper sulphate solution at the tap especially on the tongue of the plate where the plate met the surface between the air and the solution. Rinse in water containing a few drops of sulphuric acid to the liter. Dry by means of alcohol as before and weigh.

Solution for Use in Voltameter Cell. Make up a half-saturated solution of copper sulphate in water (specific gravity about 1.15) to which about one percent by volume of strong sulphuric acid has been added. After this has been used a few times it should be put in a jar for waste copper sulphate and a fresh solution made up.

Experiment 39. Determination of H at the Center of

the Coil of a Tangent Galvanometer. The tangent galvanometer is connected with long lead wires (twisted together) to two copper voltameters and a storage cell as indicated in figure 31. The tangent galvanometer is placed some distance from the rest of the apparatus so that the needle may not be affected by currents outside the instrument. Connection is made through a commutator so that the current may be reversed through the tangent galvanometer only. The galvanometer should be set up as described on page — above. The connecting wires should be closely twisted. The copper plates are prepared as described in the preceding section. Two voltameters are used so that the gains of copper on two plates may be averaged. These gain plates are numbered on the tongues so that they may not be confused in the weighings.

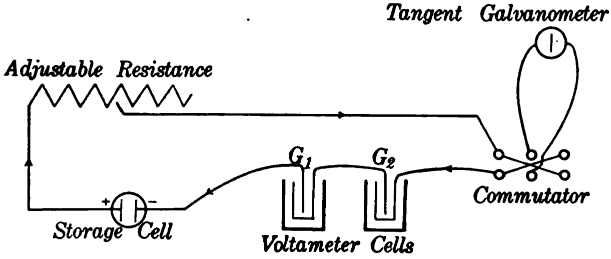


Fig. 31. Apparatus for Determining the Reduction Factor of a Tangent Galvanometer.

Two unmarked plates similar to the gain plates are provided for a preliminary test as to the setting of the adjustable resistance. Place these plates in the voltameter with the loss plates and connect up the whole circuit using 5 turns (the coil being 25-26 centimeters in diameter) on the tangent galvanometer. With the maximum value of the adjustable resistance complete the circuit at the commutator. Decrease the value of the adjustable resistance until a deflection of about 45° is obtained with the tangent galvanometer, that is a change of about 90° is made in the position of the needle when the current is reversed through the galvanometer. Leave the adjustable resistance at this value, break the current at the commutator, and substitute the weighed gain plates.

Make the current at the commutator at the beginning of a minute as indicated by your watch (the minute and second hands should have been previously made to agree). Take readings of both ends of the compass needle at the end of every minute for six minutes. Reverse the commutator at the end of this period by throwing the switch as quickly as possible, and continue the readings for another six-minute period during which the current flows through the galvanometer in the reversed direction. Take readings for five such periods in all, reversals being made at the end of each period except the last. At the end of the last period the current is broken at the commutator and the gain plates are treated as described in the preceding section. The data may be conveniently tabulated as follows (the numbers 1, 2 refer to the two ends of the needle respectively):

COMMUTATOR SWITCH	TIME			TANGENT GALVANOMETER READINGS			
	H.	M.	S.	North Pole toward East.		North Pole toward West.	
Closed at......	1	42	0	1	2	1	2
	1	43		43.7	45.2		
	1	44		43.9	45.7		
	..	..		....	....		
Reversed at...	1	48		42.6	45.1		
	1	49				44.5	43.8
	1	50				44.8	44.1
	..	..				....	....
Opened at.....	2	12	3	42.9	44.9		

Time of run, t, = 30 minutes 3 seconds — 5 seconds for reversals = 1798 seconds.

Means	43.1	45.4	44.4	43.9
Mean No. Pole E.		44.25		
Mean No. Pole W.		44.15		

Average deflection during run, θ, = 44.2°

		BEFORE RUN	AFTER RUN
Mass of Gain Plate No. 1 was	....	....	....
" " " No. 2 "	....	....	....
Gains	....	....	....

Mass of copper deposited in grams, mean of two deposits, W = ...

Quantity of electricity that has passed between electrodes,

$$Q = \frac{W}{0.000329} = \text{.................................... ......}$$

Mean current in amperes, $C = \frac{Q}{t} =$ ....

Mean radius of turns of coil, $r =$ ....

Number of turns of coil used in series, $n =$ ....

$$H = \frac{2\pi n C}{10\, r \tan\theta} =$$

The reduction factor of the galvanometer is $\frac{10\, r H}{2\pi n}$, that is the expression by which $\tan\theta$ has to be multiplied in order to get the current in amperes. H of course varies greatly with the position of the galvanometer, but the other factor depends only on the make-up of the coil and is therefore a constant of the instrument. $G = \frac{2\pi n}{r}$ is called the galvanometer constant. The current in amperes is

$$C = \frac{10\, H}{G} \cdot \tan\theta$$

III. MEASUREMENT OF CURRENT AND RESISTANCE.

An absolute measurement of electric current is one that depends on some mechanical action of the current. For example in the case of the current balance, the current passing through a coil is attracted by the same current passing through a similar coil. This attraction can be balanced by gravitational attraction and the current calculated from the amount of the attraction, the dimensions and distance apart of the attracting coils being known. A short account of this balance is given in Sylvanus Thompson's *Elementary Lessons in Electricity and Magnetism,* page 393, 1897 edition. The tangent galvanometer may also be used for absolute measurement of current if the value of H is determined by some mechanical method like those described on pages 122-129 above. By means of such methods Lord Rayleigh and others have determined with great care the value of the electro-chemical equivalent of silver with an accuracy of about one part in a thousand. In the silver voltameter the passage of one coulomb of electricity causes the deposition of 0.001118 gram of silver on the cathode. The corresponding amount of copper may be found from the atomic weights and valencies of silver and copper respectively. The atomic weights are given by Ostwald as 107.9 and 63.3. The valencies are 1 and 2 respectively. Therefore the gain of copper at the cathode after a coulomb has passed is $\frac{31.65}{107.9} \times 0.001118 = 0.000328$ gram. The corresponding numbers for oxygen and hydrogen are $\frac{1}{12050}$ and $\frac{1}{95700}$ respectively. The gas voltameter does not yield as accurate results as the silver voltameter. The amount of silver deposited on the cathode (which is usually a platinum crucible) may be weighed with great accuracy when suitable solutions are employed and when the silver anode is covered with filter paper or a porous cup. The copper voltameter, though not as much used in very accurate work as the silver voltameter, is more convenient and is capable of yielding results accurate to 0.3 percent if the instructions given on page 137 are followed. The voltameter methods of measuring mean current are more desirable for an elementary laboratory than the absolute methods.

Experiment 40. Measurement of Quantity of Electricity with the Gas Voltameter. The apparatus is arranged as in experiment 39 except that the gas voltameter is used instead of the copper voltameter. A simple form of the gas voltameter is shown in figure 32. Platinum electrodes are used because these do not unite chemically with the solution. The latter is a fifteen percent solution of sulphuric acid in water.

Use five or six storage cells and enough external resistance to give a flow of hydrogen of a few cubic centimeters a minute. If this does not give a sufficiently constant current the 110-volt circuit with a 32-candle-power lamp in series will be found to give fairly satisfactory results. The variations of current are much greater than with the copper voltameter chiefly on account of change of resistance at the electrodes where bubbles of gas collect in varying amounts. The passage of the current is opposed by the hydrogen on the cathode and by the oxygen on the anode. The electromotive force of this hydrogen-oxygen cell is very roughly about 1.5 volts. When the outside electromotive force and the outside resistance have been adjusted to give a steady flow of gas, break the circuit and draw up the liquid in each burette to near the top of the scale. This may be done by connecting to the aspirator.

It is preferable to make the calculations on the observations with the hydrogen rather than with those made on the oxygen. The volume of hydrogen is approximately twice as great. It is less soluble in the solution than oxygen. The oxygen may change to some slight extent to ozone which has only two thirds of the specific volume that oxygen has. This change takes place principally under the influence of ultra-violet light. The glass however of the burette probably does not allow any appreciable amount of this light, which is very strongly absorbed by most glass, to enter the oxygen. The flow of hydrogen should be made sufficiently slow (by regulating the external resistance) so that no appreciable amount of gas is carried (in the form of small bubbles) out of the burette by the current of liquid.

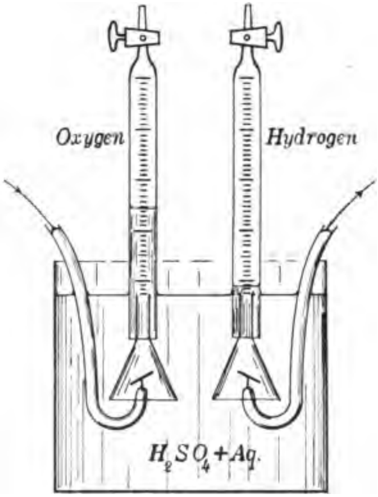


Fig. 32. Simple Form of Gas Voltameter.

Make a rough estimate of the volume included in the burette between the top of the scale and the stopcock. Read the menisci if possible to 0.1 cc.

Put down the times, the tangent galvanometer readings, and the burette readings carefully in tabular form as indicated in experiment 39.

As the acid used is dilute we may take the pressure of the water vapor over the meniscus as being equal to the vapor pressure over pure water at the same temperature. This may be obtained with sufficient accuracy by interpolation from the following:

TEMPERATURE ° C.	PRESSURE OF WATER VAPOR IN MILLIMETERS OF MERCURY
10	9.2
15	12.7
20	17.4

Let v_1 be the volume in cu. cm. of gas above the meniscus before the run. If this volume is small it appears in a small correction and it is sufficient to estimate it roughly. Let v_2 be the volume of the gas after the run. Let p_1 and p_2 be the corresponding pressures. Then the volume of the hydrogen that has gathered during the run (when reduced to the final pressure p is $v = (v_2 - v_1) + \frac{(p_2 - p_1)}{p_2} v_1$. $(v_2 - v_1)$ is here the increase in volume during the run as read off from the burette. $\frac{(p_2 - p_1)}{p_2} v_1$ is the small correction that has to be added on account of the expansion of the gas present in the burette at the beginning of the run. In estimating this small correction p_2 and p_1 may be estimated roughly by measuring the heights of the columns of liquid in the burette and subtracting from 1033 (the approximate height of the water barometer). The calculation then proceeds as follows:

Temperature of the hydrogen at the end of the run = $t°$ C = ...

Atmospheric pressure in mm. of mercury (reduced to mercury at 0° C.) = p_a = ...

Pressure of water vapor in the burette in mm. of mercury = p_w = ...

Height of meniscus above surface of liquid that meets atmosphere in mm. = h = ...

Hydrogen pressure, p_h, approximately $= p_a - p_w - \frac{h}{13.6} =$......

The mass of 1 cc. of hydrogen at 0° C. and a pressure of 760 mm. of mercury = 0.0000896 gram. = ...

The mass of v cc. of hydrogen at the above temperature and pressure is

$$m_h = v \times (.0000896) \times \frac{p_h}{760} \times \frac{273}{(273 + t)}.$$

The quantity of electricity that has passed, Q, = $m_h \times 95700$ coulombs = ...

Time of run, T = ...

The current $C = \frac{Q}{T}$ = ...

The reduction factor of the galvanometer (give number and position $= \frac{C}{\tan\theta}$ = ...

H may now be at once found if the mean radius of the number of turns used is known.

If there is time, work out the quantity, current, etc., from the amount of oxygen liberated.

Experiment 41. Absolute Measurement of Resistance. The work performed in forcing Q coulombs of electricity at a constant time rate through a given conductor is V Q, where V is the difference of potential between the terminals of the conductor. The current being constant we may write the amount of work performed, W = V Q = V.C t, where t is the time that it takes Q to pass through any equipotential section of the conductor. If there is no source of electromotive force within the conductor, we may write by Ohm's law, $V = CR$, where R is the resistance between the terminals. Therefore $W = C^2R.t$. If the conductor be surrounded by some liquid whose conductivity is very much less than that of the conductor, W may be measured by determining the heat that is given off to the liquid during the flow of Q. In order that the resistance may not change much during the experiment, manganin wire (whose temperature coefficient of resistance is only a few hundredths of a percent per degree centigrade) is used.

Place a spiral of the manganin wire (soldered at the ends to heavy copper terminals) in a calorimeter cup with about 500 grams of water.

To diminish radiation losses add small pieces of ice to the water until its temperature when thoroughly stirred is about three degrees below the temperature of the room. Use a thermometer whose scale is divided into tenths of degrees. With a reading glass, properly mounted on the tube in such a way that parallax errors are avoided, the thermometer can be read to a hundredth of a degree. The temperature is taken at regular intervals during the experiment and with especial care before the current is made and after it is broken. Make the rise of temperature about 6°.

Make the electrical connections as in experiment 39 except that the manganin spiral takes the place of the copper voltameters. Record the times, temperatures, and tangent galvanometer readings in tabular form. The water in the calorimeter vessel must be persistently stirred during the whole period of temperature readings if the indications of the thermometer are to have any value. The calculations may be conveniently made as follows:

Mass of water = ...
Water equivalent of calorimeter cup = ...
" " " " stirrer = ...
" . " " whole calorimeter = ...

Temperature just before making of current = ...
Maximum temperature reached at end of run = ...
Heat developed in calories, H = ...
Heat developed in joules, W = 4.19 H = ...
Mean current in amperes as determined by the tangent galvanometer, C = ...
Time of run in seconds, t = ...
Quantity of electricity during run, Q = C t = ...
Mean difference of potential at terminals of spiral during run, $V = \frac{W}{Q}$ = ...
Resistance of manganin spiral = $\frac{V}{C}$ = ...

Sensitive Galvanometers. Resistances can be compared with great accuracy with the aid of a sensitive galvanometer. The accuracy of comparison of resistances is much greater than the accuracy of the absolute measurement of resistance. With the aid of a number of resistances which have been accurately compared it is easy also to compare the magnitudes of steady currents. Sensitive galvanometers for steady currents may be divided into two chief types, the Thomson or the moving-magnet type, and the D'Arsonval or the moving-coil type of galvanometer.

Figure 33 shows some of the essential parts of a Thomson galvanometer. The current which is to be measured is made to pass through a very large number of turns of copper wire in series, these turns being brought as close as possible to the needle. Only one coil is indicated in the figure, but a similar coil is mounted on the opposite side of the needle so as to double the magnetic field due to the current. The deflection of the needle is read by means of a telescope and scale, the latter being reflected from a light mirror on the suspended system.

A short investigation will show the conditions for the greatest sensitiveness. We shall suppose that the coils like those indicated in figure 33 are placed so that the planes of the windings are parallel to the magnetic meridian. The suspended system will then hang with its magnets parallel to the surfaces of the coils if there is no twist in the supporting fibre and no disturbance of the earth's field. If a current C (where C is a small fraction of an ampere) be sent through the coils it will produce a magnetic field K C at right angles to the earth's field, K being the strength of field that would be produced by one ampere. Let $u\ \theta$ be the restoring couple due to the twisting of the fibre. The restoring couple due to the earth's field will be H M sin θ, where M is the magnetic moment of the suspended system. The magnetic couple due to the current is C K M cos θ. The couple due to the current's field is balanced by the couples due to the earth's field and to the twist of the fibre. Therefore $\mathrm{C\,K\,M} \cos \theta = \mathrm{H\,M} \sin \theta + u\theta$, or when θ is not more than two or three degrees, $\frac{\mathrm{C}}{\theta}\,\mathrm{K\,M} = \mathrm{H\,M} + u$, where the angle θ is expressed in circular measure. The sensitive-

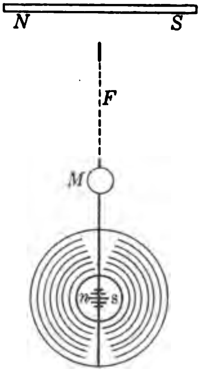


Fig. 33

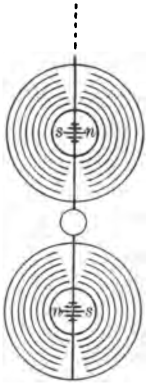


Fig. 34

Fig. 33. Thomson Galvanometer with Control Magnet to Weaken the Earth's Field.

Fig. 34. Astatic Form of Thomson Galvanometer.

ness is proportional to the angle of deflection given by the current C and is therefore measured by

$$\frac{\theta}{C} = \frac{KM}{MH + u} = \frac{K}{H + \frac{u}{M}}.$$

The sensitiveness may therefore be increased in three ways; namely by making the field due to a given current (in the coils) as great as possible, again by weakening the earth's field at the center of the coils, and lastly by making the coefficient of torsion as small as possible. The strength of the field due to a given current is increased by crowding as many windings as possible close to the suspended needle. The earth's field is weakened by means of a control magnet (placed as is for example N S in figure 33). By putting the north pole of the control magnet toward the north, the magnetic field H at the center of the coil may be reduced almost to zero. The coefficient of torsion is made small by using a very fine silk fibre or better still a very fine quartz fibre. The coefficient of torsion is also made smaller by magnetizing the small magnets of the suspended system as strongly as possible. These small magnets are made of steel hair-spring, hardened by heating to a cherry red in a copper tube and then plunging into cold water.

The magnets of the suspended system are often partially surrounded by a copper box. This serves to bring the magnets to rest quickly, the energy of rotation of the magnets being turned into heat by means of the eddy currents induced in the copper. This process is called damping. The resistance of the air also serves to diminish the amplitude of the oscillations. The measurement of the period of oscillation of the suspended system gives an indication of the strength of field H, if there be no current passing, and if the coefficient of torsion is small. If the position of the control magnet is adjusted under these conditions until the period is doubled, the sensitiveness will be increased approximately four times.

Figure 34 shows how a needle may be made approximately astatic, that is so that the earth's field will have almost no influence on the suspended system. This is the case when the magnetic moments of the upper and lower systems are equal. They are of course never exactly equal. Four coils through which the current passes in series are used.

In the *D'Arsonval galvanometer* the current is sent through a coil suspended in the field of a permanent magnet. When the instrument is in proper adjustment and no current is passing through the coil, the plane of the latter is parallel to the lines of the field and the suspending fibre has no twist. If the coil be now held while a current is sent through it, the lines of force due to the current in the coil are at right angles to the lines of force of the permanent magnet. If the field due to the latter is F lines per

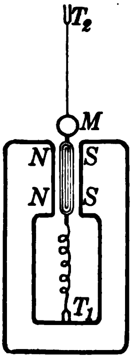


Fig. 35

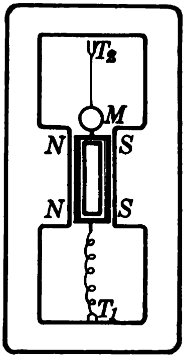


Fig. 36

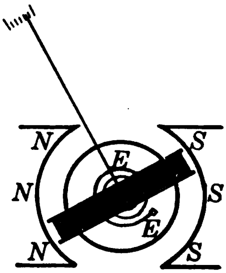


Fig. 37

Fig. 35. D'Arsonval Galvanometer.
Fig. 36. D'Arsonval Galvanometer with Yoke Magnet and Soft Iron Cylinder in Gap.
Fig. 37. Weston Ammeter.

square centimeter, if there are n turns in the coil of average area A, and if the current in C. G. S. units (1 C. G. S. unit = 10 amperes) is C, then the magnetic couple tending to turn the coil so that it will embrace more lines of the field is $F\ C\ nA$ dyne-centimeters. If the coil be released so that it turns through the angle θ and the magnetic couple is balanced by the torsional couple, then

$$F\,C\,nA \cos\theta = u\,\theta.$$

For small angles we may take $\cos\theta$ equal to 1. Then

$$\frac{\theta}{C} = \frac{F\,n\,A}{u}.$$

The condition for greatest sensitiveness is therefore that the product of the field strength of the permanent magnet and the total area of the windings of the coil should be as great as possible compared with the torsional couple per unit angle of deflection. The strength of the field in the gap is roughly inversely proportional to the width of the gap. The torsional couple produced for a given angle of deflection by a cylindrical wire of given length is proportional to the fourth power of the radius of the wire, while the weight that can be supported by the wire is proportional to the square of the radius. There is therefore great advantage in using a narrow gap in which a long narrow coil with a great number of windings of fine wire is supported by a very fine wire. If the wire is not too fine it is rolled into a ribbon. This gives a more definite zero point than a wire.

Where extreme sensitiveness is not required a wide coil is often used as in figure 36. The coil rotates around a soft iron cylinder which is fixed in the middle of the gap. With this arrangement the lines of the permanent field tend to become radial to the cylinder near its surface. The lines of force remain therefore nearly parallel to the turns of the coil as the latter rotates. The deflection is therefore more nearly proportional to the current for large deflections than is the case with the galvanometer in figure 35. If the supporting wire breaks, the coil falls on the iron cylinder and is not injured. When the instrument is not in use, the coil is let down upon the cylinder. This arrangement makes the galvanometer (of figure 36) more portable than the one indicated in figure 35.

The D'Arsonval suspension often has a single turn of copper wire parallel to the rest of the turns but insulated from them and forming a closed circuit. This very quickly brings the needle to rest, because when the coil is oscillating alternating currents are set up in the copper ring. The energy of rotation of the coil is thus quickly turned into heat by the induced currents in the copper ring. When the coil comes to its position of equilibrium without oscillating it is said to be deadbeat.

In using the sensitive galvanometers that have been described the student should be careful at first to shunt the instrument by means of a small resistance across its terminals until he is sure that he will not send too great a current through it. He should not use greater deflections than a few degrees. It is a safe rule to use the smallest current that will allow accurate measurement. This also saves the resistance boxes from undue heating by the current.

Figure 37 indicates the essential parts of a Weston ammeter. The coil swings on a pivot suspension around a soft iron cylinder in the gap of a permanent magnet. The field is radial therefore as in figure 36. The motion of the coil is resisted by the spring E. The deflections are read by means of the long pointer and a scale. Since the field of the magnet is always nearly parallel to the coil the deflections are very nearly proportional to the current. The instrument is exceedingly portable. It should be handled however as carefully as a watch.

IV. COMPARISON OF RESISTANCES.

The present practical unit of resistance is the resistance of a column of mercury 1.063 meters long and one square millimeter in cross-section, the mercury being at a temperature of 0° C. Mercury is used in the definition of the standard because it is easily obtained in a state of purity. It is very difficult on the other hand to obtain other metals in a state of sufficient purity to obtain a definite specific resistance for the metal. Hard-drawn copper has a very different specific resistance from annealed copper. Thus the specific resistance depends not only on the composition of the metal and its temperature, but also on the mechanical treatment it has undergone. When a resistance coil has been made of manganin wire for example by comparison with a standard coil it may be kept at a nearly constant resistance for years if proper precautions are taken.

Care of Resistance Boxes. Not more than a twentieth of an ampere should be sent through any of the coils that have been mounted in boxes. Resistances that may be used with large currents are mounted on special frames and the heat is taken off by air cooling or by immersing in a liquid. In the experiments which follow the resistance boxes may be most easily protected by introducing a spiral of fine German silver wire in series with the battery, so that the current through any of the coils in the box will be less than one twentieth of an ampere. The danger of injury from heating of the coils is also diminished by having a key in the battery circuit so that the current is only used during the time necessary to make the reading.

The accuracy of resistance measurements with a box of coils depends greatly on the way the plugs fit in the gaps.

The plugs of any one box should not be mixed up with the plugs of any other box. When out the plugs should be laid on the insulated top of the box. The parts of the plugs that are used to make contact should not be touched with the fingers. They should be cleaned occasionally with rouge paper or better still with chamois leather and clean infusorial earth. Just before a measurement the plugs are pressed in strongly and at the same time turned round a little. After use the plugs are loosened, as otherwise if subjected to great changes of temperature for some time the blocks (in which they fit) may be sprung.

It is very desirable to keep the top of the box clean, especially between the plugs.

Experiment 42. Comparison of Resistances by means of the Wheatstone Bridge. Specific Resistance. The theory of the Wheatstone bridge is explained in Watson, pp. 702-704. The particular form of Wheatstone bridge used in this experiment is called a meter bridge, because the arms a and b (see fig. 38) of the bridge are a uniform wire 100 centimeters long. The other two arms of the bridge are respectively the resistance r_x (which is to be determined) with connecting wires and a resistance box r with connecting wires. If we denote the resistances of the connecting wires in these two cases by c' and c respectively, we have (when r is adjusted so that no current flows through the galvanometer) $\frac{r_x + c'}{r + c} = \frac{a}{b} = \frac{a}{100 - a}$, in the case where the bridge wire is 100 centimeters long, and a is read in centimeters. The junctions of the four arms of the bridge are denoted by 1, 2, 3, 4. These may be called the four corners of the bridge. It will be noticed that the battery is connected to two opposite corners of the bridge and the galvanometer to the other two opposite corners.

The above expression for the relation among the resistances is conditioned on there being no source of electromotive force in any of the branches except the battery branch. Such electromotive forces arise for example from differences of temperature at the two ends of a copper wire which is used to connect the brass blocks (which are generally used at the terminals of the bridge wire and in the resistance boxes). The bridge wire itself is frequently made of German silver, the box coils are usually made of German silver wire or of manganin wire. To eliminate the errors due to thermal electromotive forces in the various branches it is advisable to keep the galvanometer connected all the time. The needle will then in general show a slight deflection when the battery key is open. The Wheatstone bridge adjustment consists however in seeing whether the closing of the key produces any persistent change in the deflection of the galvanometer. The term persistent is used to exclude the "kick" of the galvanometer suspension that is noticed when one of the arms of the bridge is a solenoid for example. The back electromotive force due to the building up of the magnetic field is very transient, while thermal electromotive forces are very persistent. The fol-

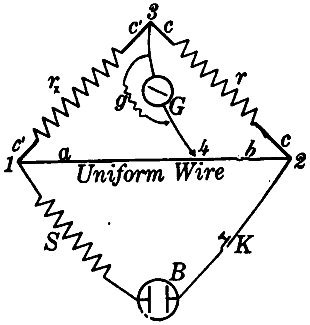


Fig. 38. Wheatstone Bridge.

lowing procedure will therefore nearly eliminate the errors due to both of these causes.

Take a galvanometer reading with all the branches of the bridge closed except the battery branch which is open at K. Close K and see whether there is any change in galvanometer deflection that persists more than a few seconds. Adjust *r* until there is no persistent change of the deflection when K is closed.

The resistance S is put in the battery branch so that the smallest currents that can be conveniently used, pass through the arms of the bridge. The galvanometer may be shunted with a resistance *g* until the final adjustment is near.

The student may convince himself of the reasons for the rule given above by placing the finger on one of the copper-brass junctions in the bridge, while he observes the effect on the galvanometer deflection. It will be found that no appreciable effect is produced by introducing such a temperature difference in the battery branch, the thermal electromotive force being small in comparison with the electromotive force of the battery. Moreover when balance is obtained the galvanometer is not affected by a change in the E. M. F. of the battery branch. To observe the effect of self-induction in one of the arms of the bridge, he may wrap one of the connecting wires, *c* or *c′*, around a piece of soft iron and then press the battery key.

The resistance of the copper connecting wires (more commonly used in this laboratory) may be obtained approximately from the following table.

NUMBER (BROWN & SHARP GAUGE)	DIAMETER IN MILLIMETERS	RESISTANCE OHMS PER METER
18	1.025	0.02
16	1.29	0.012
14	1.63	0.0076
12	2.06	0.005

It is evident from the arrangement of the Wheatstone bridge that the results are quite independent of the resistances of the connecting wires in the battery and galvanometer branches, except in the unlikely case that the resistance of these wires should be great enough to seriously reduce the deflection of the galvanometer. This would not affect the balancing point on the bridge wire, but only the precision with which the observer could determine the balancing point. If mercury cup connections are used the solid metal surfaces which are to be in contact with the mercury should be well amalgamated by washing them with a solution of nitrate of mercury. The latter should then be well removed with clean water, and the latter wiped off with a cloth.

The student is asked to measure (with a Wheatstone bridge and a reliable resistance box) resistances of the following character: a long column of mercury in a glass tube whose internal diameter is 1—2 mm., a spiral of about 10 meters of copper wire about 0.5 mm. diameter, a spiral of about 5 meters of German silver wire of about 1 mm. diameter.

The results may be conveniently exhibited as in the following table:

Material	r	c	a	b	$r_x + c'$	c'	r_x	l	A	Specific Resistance $= r_x . \frac{A}{l}$ at .. °C.

Here A represents the average area of cross-section of the wire. This may be determined by testing it at various points along its length with a pair of micrometer calipers. l represents the length of the wire included in the arm of the bridge in which it is placed.

Experiment 43. Measurement of Temperature Coef-

ficient of Resistance. The resistance whose temperature coefficient is to be measured is denoted by r_x in figure 39. The wire is wound in a bifilar manner by doubling it and winding on a frame from the end of the loop. In this way the entering current runs close by the return current so that the magnetic field due to the current is excessively small.

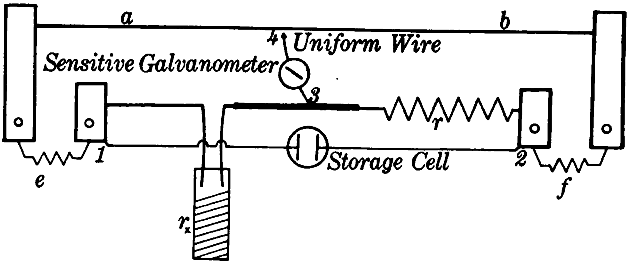


Fig. 39. Bridge for Measurement of Temperature Coefficient of Resistance.

This has also the advantage that it is easy to bring the terminals close together and therefore easy to maintain them at the same temperature.

For the sake of good insulation between the windings the resistance r_x is placed in an oil bath. For safety the latter is again heated in a large water bath. The temperature may be taken with a thermometer divided into degrees, which may be estimated to tenths. It is desirable to have the temperature range extend from 0° C. to 100° C. so that the fundamental interval $r_{100} - r_0$ may be determined. The thermometer readings should be corrected from a table of corrections given for the instrument when it is plunged in the bath up to the top of the mercury column. If part of the mercury column is exposed, the thermometer reading must be farther corrected by the addition of the term $0.00016\ n\ (t_b - t_r)$, where n is the number of exposed degrees, and t_b and t_r are the temperatures of the bath and of the air around the exposed portion of the thermometer respectively.

The electrical measurements are made as in experiment 42 except that the bridge wire is extended by means of coils of similar wire so that if e and f are the extensions of a and b respectively $\frac{r_x}{r} = \frac{a+e}{b+f}$, if the resistances of the connecting wires are so small that they may be neglected. It is desirable to adjust r to approximate equality with r_x first with only copper staples in place of e and f. The latter may then be added in the form of coils of wire which will extend the bridge wire at either end to the extent of 500-1000 centimeters.

Calibrate a copper resistance thermometer for the interval 0° C. to 100° C. Plot the observed resistances against the absolute temperature. If time permits repeat the experiment using German silver instead of copper.

The resistance of pure metals is roughly proportional to the absolute temperature. The measurement of the temperature ceofficient of resistance at low temperatures affords probably the most delicate test that we have of the freedom of a metal from alloyed material.

Experiment 44. Making of Resistance Coils with a Carey Foster Bridge. The Carey Foster bridge is a form of Wheatstone bridge used for the comparison of very nearly equal resistances. These nearly equal resistances A and B are used as extensions of the portions a and b of the bridge wire as indicated in figure 40. A commutator (see fig. 41) is provided by means of which A and B may be made to change places in the bridge arrangement. This interchange will cause a new balancing point on the bridge wire except in the special case where A and B are equal. The two portions of the bridge wire are called a' *and* b', a and a' being at the same end of the bridge wire in both cases. P and Q may be any resistances that are nearly equal. It is preferable however that they should be roughly of the same magnitude as A and B. The values of P and Q do not enter into the final calculation.

For the direct position of A and B as indicated in the diagram of figure 40, $\frac{P}{Q} = \frac{A + a}{B + b}$ and therefore

$$\frac{P + Q}{Q} = \frac{A + B + a + b}{B + b}.$$

When A and B have been interchanged and balance obtained again $\frac{P}{Q} = \frac{B + a'}{A + b'}$ and therefore

$$\frac{P + Q}{Q} = \frac{A + B + a' + b'}{A + b'}.$$

Now the numerators of the two expressions for $\frac{P + Q}{Q}$ are equal and therefore the denominators must be equal, that is $A - B = b - b'$. The difference between the two resistances that are compared by this method is therefore equal

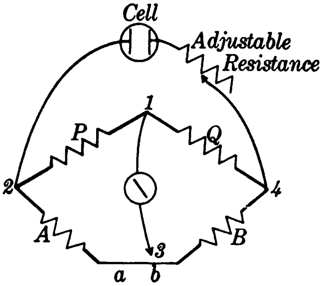


Fig. 40

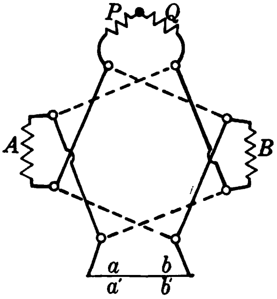


Fig. 41

Fig. 40. Carey Foster Bridge. The Resistances A and B May Be Interchanged by a Means of Commutator.

Fig. 41. Commutator Connections. $a + b = a' + b' =$ Length of Bridge Wire.

to the resistance of the portion of the bridge wire included between the two balancing points for the direct and reverse positions respectively.

To determine the resistance per centimeter of the bridge wire use in place of A a coil of small denomination, and use in place of B a short piece of heavy copper wire. If the points of balance for the direct and reverse positions are obtained on the bridge wire, the resistance of the wire lying between them is equal to that of the coil.

Make a copy of one of the coils of the standard box furnished. This copy should be made of insulated German silver wire soldered to stout copper terminals and should not differ from the original by more than a tenth of one percent.

Experiment 45. (*a*) **Measurement of Galvanometer Resistance by Thomson's Method.** (*b*) **Measurement of Battery Resistance by Mance's Method.** (*a*) The galvanometer whose resistance is to be measured is placed in one of the arms of a bridge as indicated in figure 42. Two opposite corners of the bridge are connected by means of a branch containing a cell and a resistance box *m*. The other two corners (2 and 4 in the diagram) are connected by means of a wire which may be broken at K. If the corners 2 and 4 of the bridge are at the same potential there will be no disturbance of the current through the galvanometer when the key K is opened or closed. In that case $\frac{G}{r} = \frac{b}{a}$, if the resistance of the connecting wires in the four arms can be neglected.

Start with a large resistance *m* in the battery branch and with the key K open. Adjust the resistance *m* until the deflection of the galvanometer is sufficiently large to be read to one or two percent of its total value. Then adjust *r* until the opening or closing of the key K makes no difference in the magnitude of the deflection of the galvanometer. The resistance of the latter may now be calculated by the ordinary Wheatstone bridge formula.

(*b*) The cell whose resistance is to be measured is put in the arm 2, 3 of the bridge indicated in figure 43. The arm *r* is a piece of German silver or manganin wire whose resistance per centimeter has been previously determined and whose length may be adjusted by means of binding screws at either end. B is the cell resistance.

Start with a large resistance in series with the galvanometer and adjust until a moderately large deflection of the galvanometer is obtained. Then adjust the resistance *r* until the opening or closing of the key K makes no difference in the magnitude of the deflection of the galvanometer. Then $\frac{B}{r} = \frac{b}{a}$, if the resistances of the connecting wires in the four arms of the bridge are negligible.

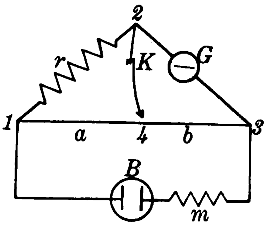


Fig. 42

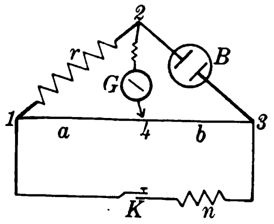


Fig. 43

Fig. 42. Bridge for Measurement of Galvanometer Resistance.
Fig. 43. Bridge for Measurement of Battery Resistance.

It is desirable to have the bridge wire of rather high resistance in this experiment as well as a resistance in series with the key K, for if the currents used are too large the battery will polarize and its resistance may change. Mance's method is not a good one for cells that polarize rapidly. In this experiment it is therefore desirable to use a comparatively constant cell such as a Daniell cell.

The Daniell cell has copper and zinc terminals which dip into a saturated solution of copper sulphate and into a solution of zinc sulphate respectively. Instead of the latter solution a 5 per cent solution of sulphuric acid may be used. There will be a great deal of local action in the latter case however unless the zinc is well amalgamated. The two solutions are separated from one another by means of a porous earthenware cup. This porous cup should be kept soaking in water when the cell is not in use. The zinc terminal should be cleaned first by scraping with a rasp and then by washing with a cloth and dilute hydrochloric acid. Lastly it is rubbed with a cloth (moistened with dilute hydrochloric acid) that has some finely divided mercury on its surface. To avoid waste of mercury the amalgamation should be performed over a large tray with a surface of several square feet.

The electromotive force remains about 1.1 volts as long as there is a bright copper surface, a saturated copper sulphate solution, and a clean amalgamated zinc surface on which no copper has been deposited. Such a deposit forms in time on account of the diffusion of the copper sulphate through the porous cup until it reaches the neighborhood of the zinc electrode. There zinc sulphate is formed and the copper deposited on the zinc. This results in local action which goes on at the surface of the zinc whether the cell is on open circuit or not. Again if the battery be short-circuited the copper sulphate around the copper electrode will be speedily exhausted. The resistance of the battery will increase and hydrogen will be deposited on the copper electrode. As hydrogen is a more oxidizable substance than copper, the electromotive force of a hydrogen-zinc cell is less than that of a copper-zinc cell. The presence of the mercury on the zinc however does not appreciatively affect the electromotive force of the cell, for the tendency of the mercury to form a sulphate is extremely small as compared with the tendency of zinc to form zinc sulphate. The zinc freely dissolves into the mercury so that a more nearly uniform surface is presented to the solution than when the zinc is not amalgamated. Local action is thus very much diminished by amalgamating the zinc.

To give a greater resistance for measurement by the Mance method, the respective solutions may be used in very small quantity. The solution in the porous cup should be about the same height as the solution outside.

Experiment 46. Measurement of Conductivity of Electrolytes with Alternating Current and Telephone. The arrangement of the bridge in this experiment differs from that used in experiment 42 (see figure 38) in that the secondary of an induction coil is used in the battery branch instead of B and a telephone is used in the galvanometer branch instead of G. No shunt is used with the telephone. One of the four arms of the bridge contains a cell like one of those indicated in figures 44 and 45. The resistance of the solution in one of these cells is called r_x. r is a resistance box whose coils are wound in a bifilar manner. a and b constitute the two portions of the bridge wire divided by the telephone terminal when the resistances of the four arms of the bridge have been so adjusted that no alternating current passes through the telephone. In that case we have $\frac{r_x}{r} = \frac{a}{b}$, if the resistances of the connections in the four arms of the bridge are negligible.

The electrodes used in the cell should be of platinum covered with a deposit of platinum black to increase the total electrode surface. (The platinizing is performed by electrolysis with a steady current through a solution of platinum tetrachloride.) The electromotive force of polarization due to the formation of products of electrolysis on the electrodes is diminished by thus increasing the electrode surface. The errors due to this source are also diminished by using a very rapidly alternating current and are smaller the greater the resistance of the electrolyte. An alternating current however is opposed by an electromotive force of self-induction in any of the arms where there are wires wound in the form of a solenoid for example. The electromotive force of self-induction is less important when the resistances of the arms in which it occurs are large. The errors due to self-induction can however be made extremely small by winding the resistance coils in a bifilar manner. With very high resistances of 10,000 ohms and above measurements with a rapidly alternating current become however difficult on account of the capacity of the resistance coils especially where they are wound closely in a bifilar manner.

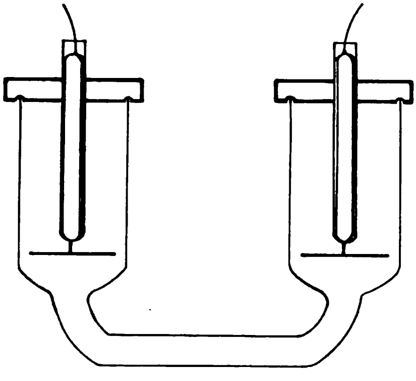


Fig. 44

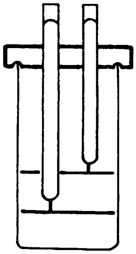


Fig. 45

Fig. 44. Cell for Concentrated Solutions of Low Specific Resistance.
Fig. 45. Cell for Dilute Solutions of High Specific Resistance.

Good measurements are therefore more likely to be made when the resistance of the electrolyte lies between 50 ohms and 3000 ohms. The platinum electrodes should be covered with a finely divided deposit of platinum. The induction coil should be regulated until a high clear note with a sharp minimum is obtained in the telephone. The induction coil should be placed in a box a long distance from the observer and should be connected with the bridge wire by means of twisted wires as far as possible.

The solutions are most easily made by diluting a concentrated solution which has been prepared by weighing out pure dry salt into a measuring flask. For example a 2 normal solution of sodium chloride is made up by weighing out 117 grams of sodium chloride (the molecular weight of sodium chloride being 58.5) and dissolving it in water in a liter flask so that a liter of uniform solution is made up. A 0.2 normal solution may now be prepared by transferring with a 25 cc. pipette (the pipette should be washed out first with a little of the solution) a corresponding amount of the 2 normal solution to a 250 cc. measuring flask and filling up the latter with water to the graduation mark. The flask should be shaken during this process and the last few drops of water added with great care. A drop of distilled water is roughly 0.05 cc. The cell and the electrodes should always be washed out with some of the solution which is to be measured before the final transfer of this solution is made to the cell. Much more care is necessary in passing from measurements of strong solutions to weak solutions than vice versa. It is desirable to keep the resistance between 50 ohms and 3000 ohms and consequently different forms of cells are used for the concentrated and dilute solutions respectively (see figures 44 and 45).

The conductivity of the solution in the given cell is the reciprocal of the resistance in the same cell. The determination of these quantities is only useful as it leads to the determination of the specific resistance and the specific conductivity, the actual dimensions of the cell depending somewhat on the accidents of glass blowing. To make the transfer from the actual resistance of any solution in the cell to the specific conductivity of the same solution, it is necessary to measure the resistance in the cell of some solution whose specific conductivity is well known. The table below gives the specific conductivities of various sodium chloride solutions for the temperature 18° C. The concentrations are given in gram-molecules per liter, unit concentration being that of a solution which has 58.5 grams of sodium chloride to a liter of solution. The specific conductivities are given in reciprocal ohms per centimeter cube. In the neighborhood of 18° C. the conductivity increases about 2 percent per degree centigrade with rise of temperature. The specific molecular conductivity is obtained by dividing the specific conductivity by the concentration. The molecular conductivity approaches a limiting value as the concentration is decreased. This is because the sodium chloride molecules are all broken up at very great dilutions so that all the sodium atoms become positive ions and all the chlorine atoms negative ions. The fraction of the sodium chloride molecules that are thus broken up is approximately obtained by dividing the values of the molecular conductivity by the maximum limiting value of the same, namely 0.1093. These results are given in the fourth column.

SOLUTIONS OF SODIUM CHLORIDE IN WATER AT 18° C.

F. KOHLRAUSCH.

Concentration Gram-Molecules per Liter	Specific Conductivity, Ohms^{-1} per Centimeter Cube	Molecular Conductivity	Degree of Dissociation
5.0	0.2115	0.0423	....
3.0	0.1683	0.0561	0.51
1.0	0.0739	0.0739	0.69
0.5	0.0402	0·0804	0.74
0.1	0.00919	0.0919	0.84
0.05	0.00476	0.0953	0.87
0.01	0.001023	0.1023	0.94
0.001	0.0001071	0.1071	0·98
0.0001	0.00001094	0.1094	1
0.00002	0.000002186	0.1093	1

If now the resistance of a normal solution be measured in the same cell that the resistance of any other solution x has also been measured, we have

$$\frac{\text{Resistance Normal Solution in Given Cell}}{\text{Resistance Solution } x \text{ in Same Cell}} = \frac{\text{Specific Conductivity of Solution } x}{0.0739},$$

where the values of the resistances are reduced to 18° C. The resistance decreases about 2 percent per degree centigrade rise of temperature. The solving of the above equation then gives the specific conductivity of the solution x at 18° C.

Measure the specific conductivities at 18° C. of as many as possible of the following solutions of sodium chloride, the numbers given expressing their concentrations in gram-molecules per liter: 2.0, 0.8, 0.4, 0.2, 0.08, 0.04, 0.02. In making the reduction to specific conductivity with the aid of Kohlrausch's values it is desirable to measure more than one of the solutions for which his values are given. If for example the student is determining the specific conductivities for various solutions between 0.1 N. and 0.01 N., it is desirable to measure in the same cell the solution 0.05 N., for which the specific conductivity is 0.00476. It would probably be better also to measure such solutions in the type of cell shown in figure 45, the rule being to use the type of cell that keeps the resistance between 50 and 3000 ohms.

Make out a table of results for the solutions that you have measured with the same headings as are used in the table above.

Experiment 47. Tests on Water by means of Conductivity Measurements. Such measurements as were described under the last experiment afford one of the most delicate tests of the freedom of water from electrolytes, that is from the acids, salts, and bases that are chemically active in water solution.

It is necessary in this experiment to use a cell like that shown in figure 45. The electrodes are placed within a few millimeters of each other. For calculation of the resistances of the different samples of water a 0.001 normal sodium chloride solution (made up with the best distilled water in the laboratory) may be measured in the cell. In passing from one specimen of water to another (and especially after the 0.001 normal solution has been used great care should be used in rinsing the cell and the electrodes with the next liquid that is to be tested.

Determine the specific conductivity for the temperature 18° C. of the tap water in the laboratory, and of specimens of water which have been distilled in different ways.

Experiment 48. Measurement of Very Low Resistances. In this experiment the difference of potential between two points on a metal rod through which a large current is flowing is balanced against the difference of potential between two points on a fine wire through which a small current is flowing. If the respective currents be C and C′ and the corresponding resistances be r and r', we have $Cr = C'r'$, from which r can be determined if the other three quantities are known.

The arrangement of apparatus is shown in figure 46. The lower circuit contains a single storage cell of very low internal resistance. Its electromotive force may be measured with a voltmeter. Let it be E′. For finding C′ the resistance of the lower circuit may be taken with sufficient accuracy as equal to the large resistance indicated in the figure, 2000 ohms for example. In that case $C' = \frac{E'}{2000}$ when there is no current passing between the upper and lower circuits. The resistance r' is that of a portion l of a uniform wire in the lower circuit. The resistance of this wire per centimeter should be known.

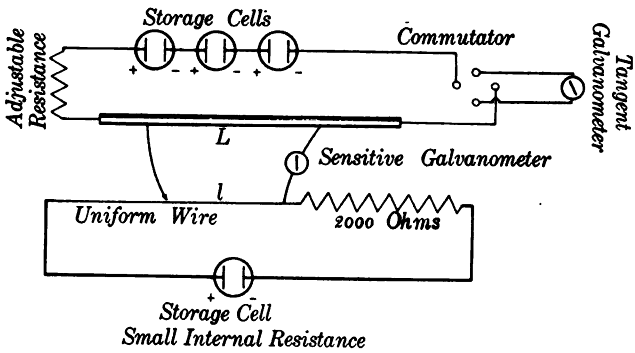


Fig. 46. Arrangement of Apparatus for Measurement of Low Resistance.

The upper circuit contains a battery of storage cells, the metal rod, an adjustable resistance, and a tangent galvanometer. The latter instrument should be a long way from the rod and should be connected to the commutator by means of long leads of wires twisted closely together so that the needle will not be appreciably affected by currents outside of the instrument. Let the current measured by the tangent galvanometer be C.

A wire is stretched across from a point on the rod in the upper circuit to a point on the fine wire in the lower circuit. Another similar connection is made through a sensitive galvanometer. One of the terminals of the sensitive galvanometer is adjusted until no deflection is obtained, indicating that the drop of potential between the two points on the rod is equal to the drop of potential between the two points on the fine wire. Let the lengths included between the points on the rod and fine wire respectively be L and *l*. Then the value of r' may be found by multiplying *l* by the resistance of the fine wire per centimeter of its length. r is found from the relation $r = \frac{C'r'}{C} = \frac{E'}{2000} \cdot \frac{r'}{C}$. The specific resistance of the material of the rod is $\frac{r.A}{L}$, where A is the average cross-section of the rod along the length L.

Determine the specific resistances of the specimens of copper, aluminum, and iron that have been furnished for this experiment.

Experiment 49. Measurement of Very High Resistances. The Wheatstone bridge does not give very satisfactory results when the resistance to be determined is a million ohms or more. If a large resistance is compared with a small resistance the balance is thrown toward one end of the bridge, where errors in determining the balancing point mount up largely. If the unknown large resistance is compared with a known large resistance, the amount of current available for the galvanometer branch of the bridge becomes very small. If the unknown resistance is so large that a storage cell in series with it and with the most sensitive galvanometer in the laboratory produces only a few centimeters deflection the Wheatstone bridge method is clearly inapplicable unless a large number of storage cells are used in series. When great differences of potential are used precautions have to be taken to prevent leakage of electricity to the ground. Without such precautions the resistance measurements would be probably quite worthless. The precautions taken in such a case are similar to those used in the electrostatic measurements described in an earlier part of this book. We shall avoid the use of such high potentials by confining ourselves to the measurement of resistances which are not more than a few megohms.

For a high resistance use a lead pencil mark on a piece of ground glass. This mark may be connected at either end with two large areas that have been blackened with the pencil. Several thicknesses of tin foil are pressed down on each of these larger surfaces by means of weights that press plane polished brass surfaces on the tin foil. The portions of the ground glass surface outside the mounted resistance are wiped with a cloth that has been dipped in kerosene. High resistances may also be made by burning a streak of "platin glanz" on a glass tube, the streak being made wider and thicker at the two ends. Some liquids, such as amyl alcohol, make fairly satisfactory high resistances.

The high resistance is connected in series with a storage cell, a very sensitive galvanometer, and a commutator. By means of the latter the storage cell is reversed with respect to the rest of the circuit and the deflection of the sensitive galvanometer noted. Let this be D. To calculate the value of the high resistance it only remains to measure the sensitiveness of the galvanometer. In order to determine this it is convenient to know the resistance of the galvanometer. With the high resistance still in series with the galvanometer shunt the latter with a resistance which reduces the deflection to one half of its previous value. Since the high resistance is very large as compared with that of the galvanometer the shunting of the latter does not appreciably affect the current through the high resistance. In this case the shunt which reduces the deflection of the galvanometer to one half of its previous value must have a resistance equal to that of the galvanometer itself.

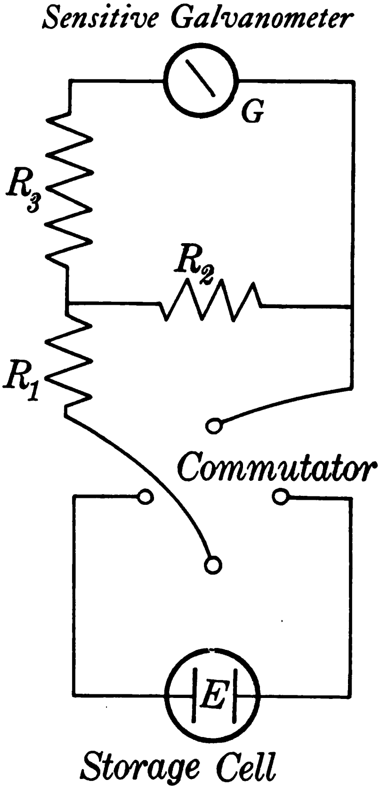


Fig. 47. Arrangement for Calibration of Sensitive Galvanometer.

The sensitiveness of the galvanometer (with the same scale distance as before) may now be determined by the arrangement shown in figure 47. The same storage cell that was used before is connected through a commutator in series with a resistance R_1 (about 500 ohms) and a small resistance R_2 of about one ohm. The latter is shunted with a large resistance R_3, which is adjusted until the deflection of the galvanometer G is on the scale. Call the deflection of the galvanometer (obtained by reversing the storage cell at K) D′ centimeters. Let the electromotive force of the storage cell as measured with the voltmeter be E. Then the current through the galvanometer is approximately $C = \frac{E}{(R_1 + R_2)} \cdot \frac{R_2}{(R_3 + G)}$, R_2 being supposed to be very small in comparison with R_3. The current which will give a deflection of 1 centimeter on reversal (for the given scale distance) is therefore $\frac{C}{D'}$.

If R be the value of the high resistance which is to be determined, we have

$$\frac{E}{R + G} = \frac{C.D}{D'}.$$

The value of R may now be determined.

QUESTIONS

1. Give the mean radius of a tangent galvanometer coil, which when placed in the magnetic meridian will deflect a magnetic needle 45° when a current of 0.01 ampere is sent through 10 turns in series.

2. How much heat will be developed per hour in 10 meters of German silver wire (of specific resistance 30 millionths of an ohm) 1 mm. in diameter by a current of 10 amperes?

3. Describe an apparatus for measuring current whose indications are proportional to the square of the current.

4. Show how a current of the order of a thousandth of an ampere may be accurately measured if one has a standard cell, good resistance boxes and a sensitive galvanometer.

V. MEASUREMENT OF ELECTROMOTIVE FORCES.

The electromotive force of a cell may be defined as the difference of potential that the cell can maintain between two terminal wires composed of the same metal and at the same temperature. If the terminal wires were connected to an electrometer, the electromotive force of the cell could be thus measured on open circuit.

In this laboratory the most exact method of determining the E. M. F. of a cell is by comparison with the E. M. F. of a standard cell. The latter is a cell that does not polarize appreciably (when used with small currents), and has a small temperature coefficient. The Daniell cell which has already been described was one of the earliest of these cells. The Clark cell was the standard for a long time. The positive electrode was mercury, the negative electrode an amalgam containing 10 percent of zinc and 90 percent of mercury. The former was in contact with mercurous sulphate, the latter in contact with a saturated solution of zinc sulphate, the solution of zinc sulphate being in contact with the paste of mercurous sulphate. The E. M. F. of the Clark cell is $\{1.4328 - 0.00119\,(t^\circ - 15^\circ)\}$ volts. The temperature coefficient is thus rather large for exact work.

The cadmium cell is made up in a manner very similar to that used with the Clark cell, cadmium taking the place of zinc in the amalgam and in the sulphate of the saturated solution. The proportion of the cadmium in the amalgam is however from 12 to 14 percent. The arrangement is shown in figure 48. Full directions for setting up the cell will be found in Ostwald-Luther, *Physiko-Chemische Messungen,* pp. 361-364. The E. M. F. of the cell thus made up is $\{1.0186 - 0.00004\,(t^\circ - 20^\circ)\}$ volts, when currents of not more than 0.0001 ampere are sent through it.

Various cells may be compared with the cadmium standard cell by means of the following arrangement known as the potentiometer.

Experiment 50. Comparison of Electromotive Forces by means of the Potentiometer. The arrangement for the comparison of cells is shown in figure 49. The lower circuit contains a storage cell, an adjustable resistance R′, and a uniform wire. The E. M. F. of the storage cell must be greater than that of any of the cells which are to be compared with each other. If necessary two or more storage cells may be used. The resistance R′ is adjusted so that the difference of potential between the ends of the uniform wire is greater than any of the cells to be tested. During the actual comparison R′ must be maintained constant as it is necessary that the current C should be maintained constant during the comparison of the cells.

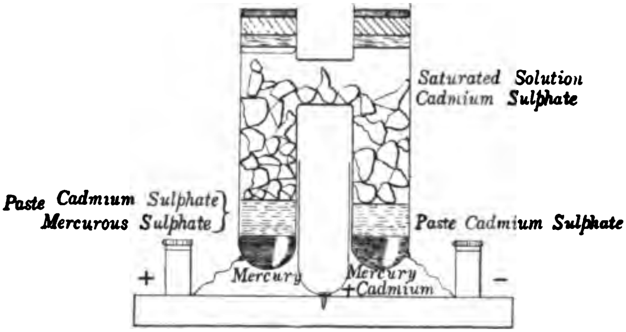


Fig. 48. Cadmium Cell.

Each of the cells to be tested may be put in series with a high resistance R″ and a sensitive galvanometer. By means of the terminals A and Y connection is made with the uniform wire. When the difference of potential between A and Y is equal to the E. M. F. of the cell in series with the galvanometer, no current will flow through the latter. By means of the commutator K the standard cell may be substituted for the one with which it is being compared. If the points of balance obtained in the two cases are Y_x and Y_s respectively, while the corresponding values of the E. M. F. are E_x and E_s, then

$$\frac{E_x}{E_s} = \frac{(AY_x)}{(AY_s)}.$$

The ratio of the electromotive forces may thus be obtained from the ratio of two lengths.

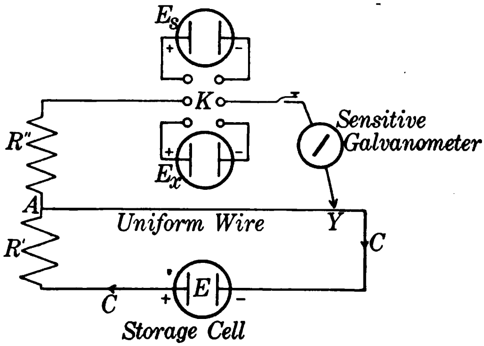


Fig. 49. Potentiometer for the Comparison of the Electromotive Forces of Two Cells.

The value of the resistance R″ does not come into the calculation. It is important to have it as large as possible however so that too large currents may not be sent through the sensitive galvanometer or the cells which are being compared. This might be done before the correct length for balance had been ascertained. It is better therefore to start with R″ very large. It may be diminished as the point of balance is approached.

The following exercises with the potentiometer are recommended:

(*a*) Compare the electromotive forces of a Daniell cell and of a LeClanché cell with a cadmium cell. Report the temperatures of the two cells first mentioned.

(*b*) Compare the E. M. F. of a given Daniell cell with the difference of potential at its terminals when they are joined by an external resistance r. If the internal resistance of the cell be b, and the above difference of potential at the terminals be D, then

$$\frac{D}{E} = \frac{r}{r + b}.$$

Hence calculate b. The resistance r should not be taken from a resistance-box, but may be a resistance of one or two ohms made of a piece of manganin wire soldered to copper terminals.

Experiment 51. Measurements of Thermal Electromotive Forces. The object of this experiment is to ascertain how the thermal E. M. F. of a copper-iron thermocouple varies with the temperature of the hotter junction, the colder junction being kept at 0° C. A simple arrangement for balancing the E. M. F. of the couple is shown in figure 50. For the sake of simplicity in the figure only one copper-iron couple is shown, but it is desirable to use a number of similar couples in series in order to multiply the E. M. F. to be measured. The resistance R′ may be conveniently 200-300 ohms and the resistance of the uniform wire about 0.5 ohm per meter. The value of R′ may be read from the resistance-box used for the purpose. The resistance of the uniform wire per centimeter should be known as accurately as possible.

As in the last experiment the point of contact Y of one of the terminals of the couple is adjusted until there is no current through the sensitive galvanometer. If R be the resistance of the length of wire AY when balance has been obtained, the thermal electromotive force, $E_t = C\ R$, where C is the current through the storage cell. This current may be determined by measuring the E. M. F. of the cell with

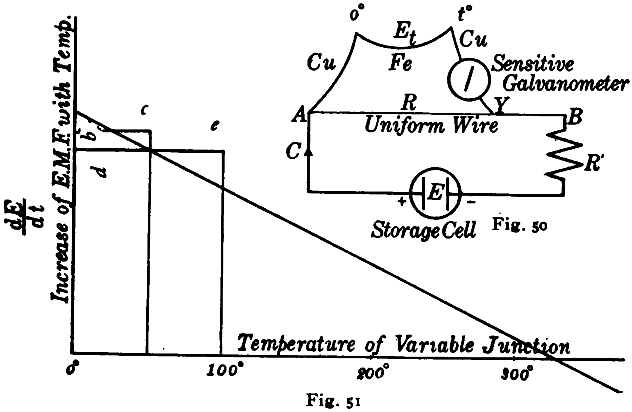


Fig. 51

Fig. 50. Arrangement for Measurement of Electromotive-Force of Thermocouple.

Fig. 51. Thermoelectric Diagram for Finding the Neutral Point of a Copper-Iron Couple.

the voltmeter and dividing by the total resistance through which the current C flows. Since none of it flows through the thermocouple this resistance is equal to $R' + R_{AB}$, where R_{AB} is the resistance of the uniform wire. The resistance of the storage cell should be so small as to be negligible. Thus $C = \frac{E}{R' + R_{AB}}$ and $E_t = \frac{R\ .\ E}{R' + R_{AB}}$.

Report the results for one copper-iron couple as indicated below. Plot six to eight sets of readings from 0° to 100°.

Temperature of Hot Junction, °C.	Temperature Cold Junction, approximately 0° C.	Thermal Electromotive Force in Volts.

An approximate thermoelectric diagram for iron with respect to copper may be constructed as follows: Lay off as in figure 51 a horizontal line representing temperatures. On the line 0, 50 erect a rectangle O, *b, c,* 50 of any convenient height. We shall arbitrarily take the area of this rectangle as representing the E. M. F., E_{50}. On the line 0, 100 erect a rectangle *o, d, e,* 100 of such a height that its area shall represent on the same scale as before the E. M. F. E_{100}. Since E_{100} is not twice as great as E_{50} the height of the second rectangle will be less twice that of the first rectangle. Join the middle points of *bc* and *de* and produce the line so obtained to cut the axis of temperatures. If the measurements have been well performed the point of intersection will give the temperature of the neutral point. At this temperature for the hot junction (the cold junction being kept at 0° C.) the thermal electromotive force is a maximum. If the temperature of the hot junction be carried still higher, the E. M. F. will decrease till it reaches the value zero and then becomes reversed.

Thermocouples are much used in high temperature measurements, especially a couple of which one member is platinum wire while the other is a wire drawn from an alloy containing 90 percent of platinum and 10 percent of rhodium. This couple may be used up to 1600° C. It shows a continuous increase of E. M. F. as the temperature rises, although the increase of E. M. F. is not proportional to the increase of temperature. The methods of calibrating such couples may be studied in an advanced course specially devoted to heat measurements.

Causes of Inconstancy of Electromotive Force in Galvanic Cells. The electromotive force of a galvanic cell depends not only on the particular metals used as electrodes but also on the solutions which surround these electrodes, and on the temperature. In a Daniell cell for example there is on open circuit a potential difference of about half a volt between the copper electrode and the solution of copper sulphate. There is a potential difference of nearly similar amount but in the opposite direction between the zinc and zinc sulphate solution. There is also a small jump of potential at the boundaries of the copper sulphate and zinc sulphate solutions. These three potential differences add up to about 1.1 volts. As long as only a small current is sent through the cell copper is deposited from the saturated copper sulphate solution on to the copper electrode and the other changes are such as to leave the composition of the electrodes and of the solutions unaltered. If however the cell be short-circuited the copper is more rapidly exhausted from the neighborhood of the copper electrode than it can be supplied by diffusion of copper sulphate. Then hydrogen begins to be deposited on the electrode instead of copper and the E. M. F. of the cell decreases. If the large current is kept up the E. M. F. will fall in time to about 0.6 volt.

Various devices are used to absorb hydrogen in various types of cell. This absorption diminishes the polarization and so retards the running down of the cell. In the LeClanché cell for example manganese peroxide is packed around the carbon electrode. This disposes of a portion of the hydrogen by oxidizing it. The E. M. F. of the cell consequently does not fall off so rapidly as it would if the manganese dioxide were not present. Chromic acid and nitric acids are used in some other forms of cells for a similar purpose.

Experiment 52. Measurements of Electromotive Force of Polarization. Platinum electrodes are used in a vessel (called Z in figure 52) containing a dilute solution of sulphuric acid in water. It is better to have the electrodes platinized than to use bright platinum. The cell Z is charged by connecting it to the battery B. The battery B′ in the upper circuit is chosen so that the difference of potential between *a* and *b* is more than two volts. The cell Z is now disconnected from B by throwing the double switch K over to the position where it makes contact between Z and the potentiometer wire. At the start Y is near *b*. It is moved quickly toward *b* to the first position for which no current passes through the galvanometer G. The charging and balancing on the potentiometer wire are repeated until the maximum length *a*Y (for which no current will flow through G) is obtained. It will be found that if the point of contact is moved over toward *a* the current through G again becomes zero. The E. M. F. of polarization in this case prevents the sending of a continuous current (unless it be a very slight leakage) through the cell. If the point Y be moved however toward *b* until the difference of potential is greater than the maximum E. M. F. of polarization, a continuous current will be forced through the cell Z by the battery B′.

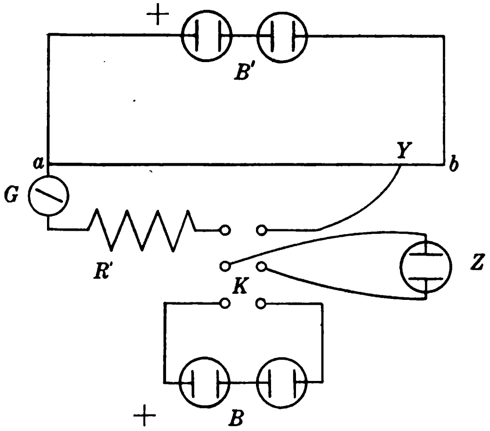


Fig. 52. Arrangement for Measurement of Maximum Electromotive Force of Polarization.

Calculate by one of the methods used in the last two experiments the maximum E. M. F. of polarization for platinum plates charged in dilute sulphuric acid.

Repeat the experiment with lead plates in a solution of sulphuric acid in water, the density of the solution being about 1.15.

Use of Commercial Storage Cell. The cell as made in the above exercise from solid lead plates has not sufficient capacity for commercial use. Lead grids are therefore cast with a large number of openings. These are packed with spongy lead or lead sulphate or lead oxide. When the cell is charged by passing a current through it, the plate by which the current enters becomes coated with lead peroxide. Spongy lead is formed on the plate by which the current leaves the cell. The reaction may very probably be represented by the formula

$$PbO_2 + Pb + 2\ H_2SO_4 \rightleftarrows 2\ PbSO_4 + 2\ H_2O.$$

During the discharge the reaction takes place from left to right, while during the charging it takes place from right to left. Thus during the discharge the red peroxide of lead is broken down to sulphate of lead and the sulphuric acid becomes less concentrated. For example in some storage cells the specific gravity of the acid when freshly charged is about 1.18. During discharge this falls to about 1.15.

The charging or discharging currents should not be greater than 0.01 ampere per square centimeter of outside surface (not counting convolutions) of the positive plates used in parallel. Very rapid discharge uses up the sulphuric acid in the pores of the plates too quickly, thus increasing the internal resistance of the cell and diminishing the difference of potential at the terminals of the cell. If it be allowed to rest after use the sulphuric acid from the outside passes into the pores by diffusion and so the cell gradually recovers. A direct-current dynamo is used to charge the cells in this laboratory, the positive terminal of the dynamo being connected to the positive pole of the battery, while the negative terminal of the dynamo is connected to the negative terminal of the battery through an ammeter and sufficient resistance to keep the current below the limit given above. The charging is kept up until the plates give off considerable quantities of gas. To prevent splashing of the acid the surface of the acid solution is covered with a layer of paraffin oil. If the surface of the acid falls sufficiently to uncover the plates the cell should be filled up again with distilled water. It is desirable above all to avoid the presence of iron or of iron salts in the cell.

Experiment 53. Electromotive Force in a Conductor Revolving in a Uniform Field. A coil containing some hundreds of turns of insulated wire is mounted so that it can be revolved around a vertical axis. Call the total area of the coil (mean area of a turn multiplied by the number of turns) by the letter A. Let the horizontal component of the earth's field be H. Then if the coil is in a vertical plane perpendicular to the magnetic meridian the number of lines of force embraced by the windings of the coil is H A. If the coil be turned to another vertical plane making the angle θ with the original position, the number of lines of force embraced by the turns of the coil is $H\ A \cos \theta$. Now suppose the coil to be revolved around its vertical axis of rotation with uniform angular velocity. The value of the electromotive force induced in the coil is equal at any instant to the time rate at which the lines of force are being cut. This is evidently greatest when the plane of the coil is turning through the magnetic meridian and becomes zero when the plane of the coil is perpendicular to the magnetic meridian. The electromotive force in the coil then reverses its sign. The upper curve in figure 53 shows the changes in the E. M. F. during the revolution of the coil. The abscissæ represents times, the ordinates the corresponding values of the E. M. F. in the coil. The time from *a* to *c* is the period of a complete revolution of the coil.

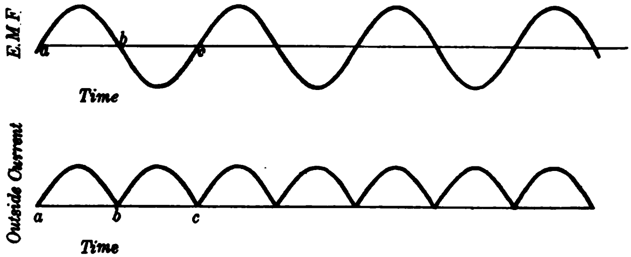


Fig. 53. The Upper Curve Represents E. M. F. in Coil Revolving in Uniform Field. The Lower Curve Indicates Current in Circuit Outside of Commutator.

The current is taken from the revolving coil by means of two brushes which press on the half cylinders of brass that form the terminals of the coil. These half cylinders are placed on the axis of rotation of the coil in such a way that each terminal changes from one brush to the opposite at the instant that the E. M. F. passes through its zero value. The current in the portion of the circuit outside the coil therefore goes through the changes indicated in the lower curve of figure 53.

The mean numerical value of the E. M. F. may be easily calculated if we reflect that the coil cuts H A lines of force in a fourth of a complete revolution. If the coil then makes *n* revolutions a second the mean E. M. F. is $4\,H\,A\,n$ C. G. S. units, that is $\frac{4\,H\,A\,n}{10^8}$ volts, from which the mean E. M. F. may be easily calculated if the number of revolutions per second be counted and the area of the coil and the value of H be known. If the resistance R of the circuit be large the mean current through a galvanometer in the outside portion of the circuit will be very nearly $\frac{4\,H\,A\,n}{R\,.\,10^8}$ amperes.

Determine the sensitiveness of the galvanometer from its deflection for a given number of revolutions of the coil per second. The values of A, R, and H will be furnished to the student who needs to know them.

Revolve the coil around a horizontal axis of rotation with about the same angular velocity as before. The ratio of the deflections of the galvanometer in the two cases gives approximately the ratio between the horizontal and vertical components of the earth's field. From these observations calculate the angle of dip at the place of the coil.

QUESTIONS

1. 30 storage cells (charging voltage 2.5 volts) are charged in series from a 110-volt circuit. What resistance will have to be used in series to keep the charging current at 10 amperes if the resistance of each cell is 0.01 ohm.

2. How can a Wheatstone bridge be adapted for use with a resistance coil in the measurement of high temperatures? How would it be calibrated? In what cases is it preferable to a thermocouple?

3. Describe a method of measuring the E. M. F. of a cell that is independent of the resistance of the cell.

4. How much copper would be deposited in a large copper voltameter by a steady current of 5 amperes flowing for one day? What volume of hydrogen at atmospheric pressure at 20° C. would be liberated from a solution of sulphuric acid by the same current at the same time?

5. A coil of 5000 turns wound in series (mean area of one turn 50 square centimeters) is turned at the rate of 1000 revolutions a minute in a field of 200 C. S. S. units. What voltage can be obtained at the terminals of the coil in open circuit?

VI. COMPARISON OF MAGNETIC FIELDS. COMPARISON OF SPECIFIC INDUCTIVE CAPACITIES.

If the earth-inductor coil be turned from a position in which it is in a vertical plane at right angles to the magnetic meridian through an angle of 180° around a vertical axis, it will cut 2 H A lines of force. Using the same notation as in the last experiment the mean E. M. F. during the rotation is $4\,H\,A\,n$ and if the coil be connected to a galvanometer the mean current is $\frac{4\,H\,A\,n}{R}$ where R is the resistance of the whole circuit. The quantity of electricity that flows through the circuit is equal to the mean current multiplied by the time of flow, that is $\left(\frac{4\,H\,A\,n}{R}\right) \cdot \left(\frac{1}{2} \cdot \frac{1}{n}\right) = \frac{2\,H\,A}{R}$ in C. G. S. units.

The transitory current will produce a throw of the galvanometer needle or coil which will for small deflections be proportional to the quantity of electricity that passes through

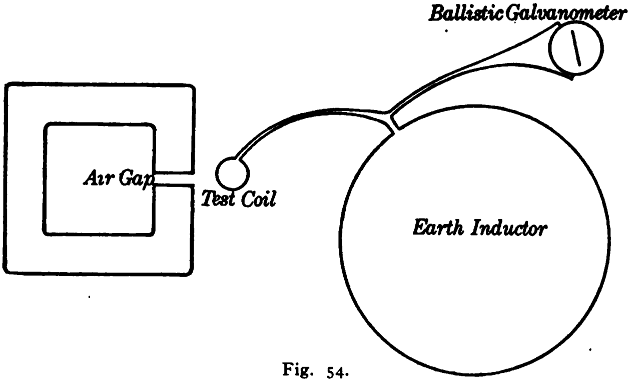


Fig. 54.

the circuit. When the galvanometer is used in this way it is called a ballistic galvanometer. It may be calibrated by noting the throws when earth-inductor coils of different areas are turned through 180° as indicated above. The resistance of the whole circuit has to be known in each case.

Experiment 54. Measurement of the Magnetic Field between the Pole Pieces of a Permanent Horseshoe Magnet. Intensity of Magnetization. The test coil is placed for example in the gap of a horse-shoe magnet so that the plane of the coil is perpendicular to the lines of force in the gap. After the galvanometer has come to rest the coil is jerked out and the deflection given by the first throw t_b read. Let the total area of the test coil be a and the number of lines of force (in the gap) per sq. cm. be B. The throw produced by turning the earth-inductor through 180° (from a position in which its axis points north and south) is also noted. Let this be t_h. Then disregarding corrections for damping:

$$\frac{t_b}{t_h} = \frac{a\,\mathrm{B}}{2\,\mathrm{HA}}$$

where A is the total area of the earth-inductor coil.

The number of lines of induction passing through a long bar magnet may be determined by placing the test coil of n turns so as to include the lines of induction at the middle of the magnet, the coil being perpendicular to the lines of induction. The throw t_m produced by jerking the coil to a position some distance from the strong magnetic field may be compared with the earth-inductor throw as above, and the number of lines of induction N calculated from the equation:

$$\frac{t_m}{t_h} = \frac{\mathrm{N}\,n}{2\,\mathrm{HA}}$$

The student may as an exercise measure B for the permanent magnet of one of the D'Arsonval galvanometers and measure the number of lines of induction N passing through the middle of a long bar magnet. If q be the area of cross-section of the iron then B is N/q. At the center of the magnet (neglecting the poles and the influence of the earth's field) B is equal to $4\pi I$, which is equal to $4\pi M/lq$, where M is the magnetic moment of the magnet, I the intensity of magnetization, and lq the volume of the magnet. Calculate the magnetic moment of the bar magnet in this way.

The earth inductor should be placed some distance from the magnets so that the field may be undisturbed by the magnet and sensibly uniform. Connections should be twisted together so that inductive action may be confined to the coils whose areas are known.

Experiment 55. Measurement of Specific Inductive Capacity. The specific inductive capacity of a given material may be measured by comparing the capacity of a condenser with fixed plates when air and the given material are used respectively as dielectrics. The arrangement of apparatus is shown in figure 55. A large number (100-200) of storage cells (well insulated from each other and the surroundings) are connected in series, one pole of

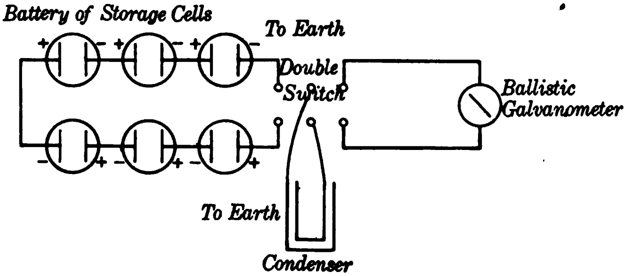


Fig. 55.

the battery being connected to the ground. The condenser plates are put in a tank so that some non-conducting liquid may be used as dielectric.

By throwing the switch to the left the condenser is charged when air is the medium. By throwing the switch to the right the condenser is discharged through the galvanometer. Let the first throw be t_a. The experiment is repeated when the dielectric in the condenser is kerosene oil. Let the throw be t_o. Then the specific inductive capacity of the kerosene oil k_o is given by the equation

$$\frac{k_o}{k_a} = \frac{t_o}{t_a}$$

The specific inductive capacity of air k_a may be taken as 1.

QUESTIONS

1. What is Maxwell's relation between specific inductive capacity and refractive index? Illustrate the truth of his relation in the case of kerosene oil.

2. What is the refractive index of water for long ether-waves? Compare this value with the square root of the specific inductive capacity.

3. Two pith-balls charged each with 100 C. G. S. electrostatic units, are distant 50 centimeters from each other first in air, then in mica. Compare the forces between them in the two cases.

4. What is the cause of the polarization in a copper, zinc, sulphuric acid cell?

5. Describe with drawings three types of cells which do not polarize with small currents.

6. What is the neutral point? What is the value of the Peltier effect at the neutral point?

EXP
McK

HW 1WY9 W

Please sign name and address on the card and leave it in the box provided.

Books are to be returned within 24 hours.